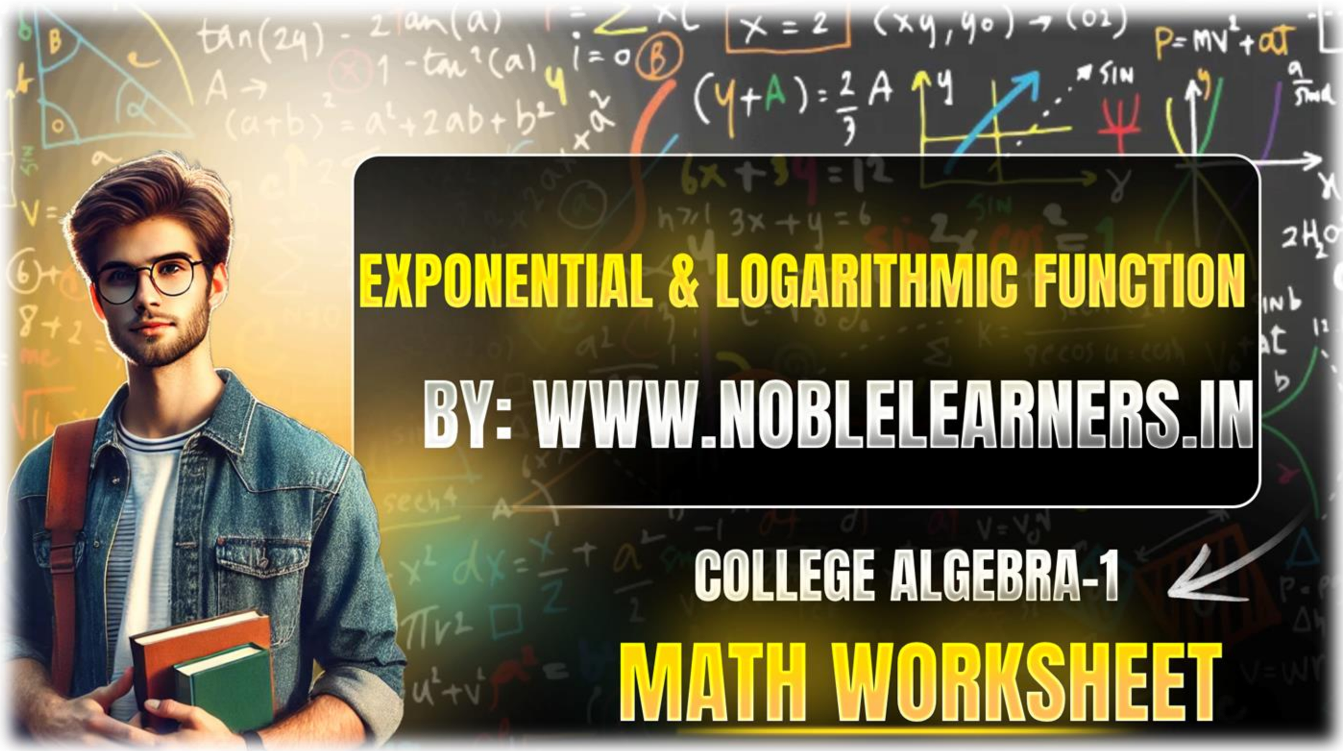

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4.1 Exercises

1. The inverse of every logarithmic function is an exponential function and vice-versa. What does this tell us about the relationship between the coordinates of the points on the graphs of each?
2. Does the graph of a logarithmic function have a horizontal asymptote? Explain.

In Exercises 3 – 8, rewrite each equation in exponential form.

3. $\log_5 25 = 2$
4. $\log_{25} 5 = \frac{1}{2}$
5. $\log_3\left(\frac{1}{81}\right) = -4$
6. $\log_{\frac{4}{3}}\left(\frac{3}{4}\right) = -1$
7. $\log_b m = c$
8. $\log_5 a = 2$

In Exercises 9 – 14, rewrite each equation in logarithmic form.

9. $2^3 = 8$
10. $5^{-3} = \frac{1}{125}$
11. $4^{\frac{5}{2}} = 32$
12. $\left(\frac{1}{3}\right)^{-2} = 9$
13. $a^b = m$
14. $b^3 = c$

In Exercises 15 – 35, simplify the expression without using a calculator.

15. $\log_3 27$
16. $\log_6 216$
17. $\log_2 32$
18. $\log_a a^6$
19. $\log_6\left(\frac{1}{36}\right)$
20. $\log_{27} 9$
21. $\log_{36} 216$
22. $\log_{\frac{1}{5}} 625$
23. $\log_{\frac{1}{6}} 216$
24. $\log_{\frac{1}{c}} c^2$
25. $\log_{36} 36$
26. $\log_4 8$
27. $\log_6 1$
28. $\log_c 1$
29. $\log_{13}\sqrt{13}$
30. $\log_{36}\sqrt[4]{36}$
31. $7^{\log_7 3}$
32. $36^{\log_{36} 216}$
33. $\log_{36} 36^{216}$
34. $\log_5 3^{\log_3 5}$
35. $\log_2 3^{\log_3 2}$

In Exercises 36 – 38, determine two integers that bound the value of the logarithmic expression; one integer that is smaller and one integer that is larger than the expression.

36. $\log_3 59$
37. $\log_4\left(\frac{1}{14}\right)$
38. $\log_{10} 900$

In Exercises 39 – 47, solve the equation using the one-to-one property of exponential functions.

39. $2^{4x}=8$

40. $3^{x-1}=27$

41. $5^{2x-1}=125$

42. $4^{2x}=\frac{1}{2}$

43. $8^{x}=\frac{1}{128}$

44. $2^{x^3-x}=1$

45. $3^{7x}=81^{4-2x}$

46. $3^{7x+2}=\left(\frac{1}{9}\right)^{2x}$

47. $\left(\frac{1}{2}\right)^{3x}=2^{x+4}$

In Exercises 48 – 56, solve the equation by converting the logarithmic equation to exponential form.

48. $\log_3 x=2$

49. $\log_2 x=6$

50. $\log_9 x=\frac{1}{2}$

51. $\log_6 x=-3$

52. $\log_2 x=-3$

53. $\log_3 x=3$

54. $\log_{18} x=2$

55. $\log_3\left(7-2x\right)=2$

56. $\log_{\frac{1}{2}}\left(2x-1\right)=-3$

In Exercises 57 – 62, sketch the graph of $y=g\left(x\right)$ by starting with the graph of $y=f\left(x\right)$ and using transformations. Track at least three points of your choice through the transformations. State the domain, range, and asymptote of g.

57. $f\left(x\right)=2^{x}$; $g\left(x\right)=2^{x}-1$

58. $f\left(x\right)=2^{x}$; $g\left(x\right)=2^{-x}$

59. $f\left(x\right)=2^{x}$; $g\left(x\right)=-2^{x}+3$

60. $f\left(x\right)=2^{x}$; $g\left(x\right)=2^{x-2}$

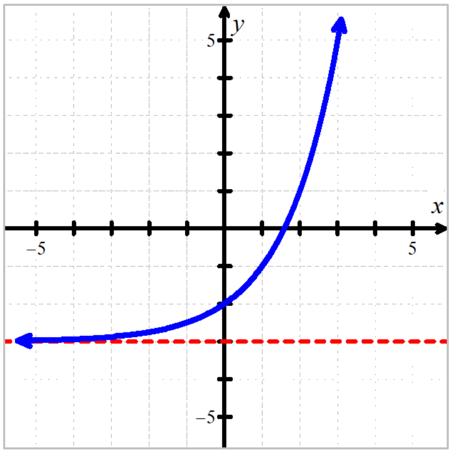
61. $f\left(x\right)=\left(\frac{1}{3}\right)^{x}$; $g\left(x\right)=\left(\frac{1}{3}\right)^{x-1}$

62. $f\left(x\right)=3^{x}$; $g\left(x\right)=3^{-x}+2$

In Exercises 63 – 66, identify a function whose graph is shown. Asymptotes are drawn as dashed lines.

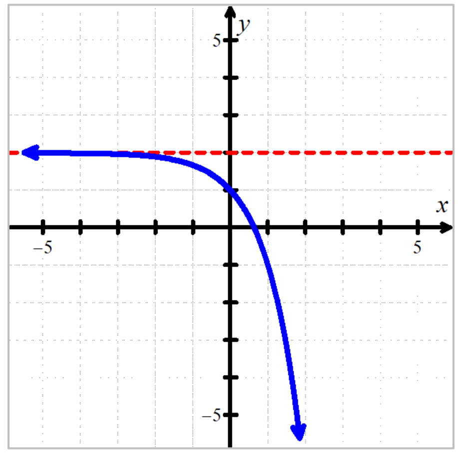
63.

Figure Ex4.1. 1



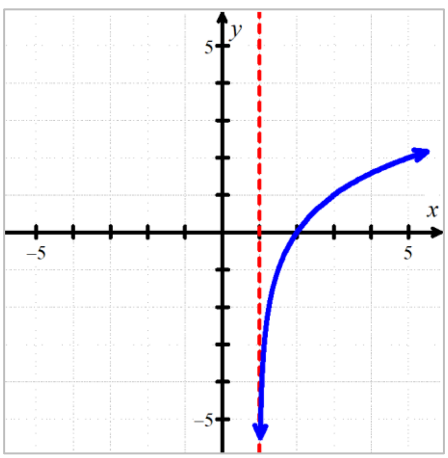
64.

Figure Ex4.1. 2



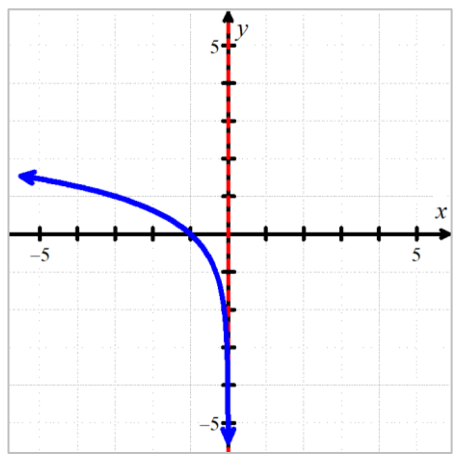
65.

Figure Ex4.1. 3



66.

Figure Ex4.1. 4



In Exercises 67 – 72, sketch the graph of $y = g(x)$ by starting with the graph of $y = f(x)$ and using transformations. Track at least three points of your choice through the transformations. State the domain, range, and asymptote of g .

67. $f(x) = \log_2 x$; $g(x) = \log_2(x+1)$

68. $f(x) = \log_2 x$; $g(x) = \log_2(x-2)$

69. $f(x) = \log_{\frac{1}{3}} x$; $g(x) = \log_{\frac{1}{3}} x + 1$

70. $f(x) = \log_3 x$; $g(x) = -\log_3(x-2)$

71. $f(x) = \log_4 x$; $g(x) = 2\log_4 x$

72. $f(x) = \log_3 x$; $g(x) = 2\log_3(x+4) - 1$

(Logarithmic Scales) In Exercises 73 – 75, we introduce three widely used measurement scales that involve logarithms: the Richter scale, the decibel scale and the pH scale. The computations involved in all three scales are nearly identical so pay attention to the subtle differences.

73. Earthquakes are complicated events and it is not our intent to provide a complete discussion of the science involved in them. Instead, we refer the interested reader to a solid course in Geology or the U.S. Geological Survey's Earthquake Hazards Program. The Richter scale measures the magnitude of an earthquake by comparing the amplitude of the seismic waves of the given earthquake to those of a 'magnitude 0 event', which was chosen to be a seismograph reading of 0.001 millimeters recorded on a seismometer 100 kilometers from the earthquake's epicenter. Specifically, the magnitude of an earthquake is given by

$$M(x) = \log_{10}\left(\frac{x}{0.001}\right)$$

where x is the seismograph reading in millimeters of the earthquake recorded 100 kilometers from the epicenter.[5]

(a) Show that $M(0.001) = 0$.

(b) Compute $M(80{,}000)$.

(c) Show that an earthquake that registered 6.7 on the Richter scale had a seismograph reading ten times larger than one that measured 5.7.

74. While the decibel scale can be used in many disciplines, we shall restrict our attention to its use in acoustics, specifically its use in measuring the intensity level of sound. The Sound Intensity Level L (measured in decibels) of a sound intensity I (measured in watts per square meter) is given by

$$L(I) = 10\log_{10}\left(\frac{I}{10^{-12}}\right).$$

Like the Richter scale, this scale compares I to a baseline: $10^{-12}\,\frac{W}{m^2}$ is the threshold of human hearing.

(a) Compute $L(10^{-6})$.

[5] To evaluate an expression of log with base 10 using a calculator, look for a 'log' key. As we will discover shortly, '$\log_{10}$' is frequently simplified as 'log'.

(b) Damage to your hearing can start with short term exposure to sound levels around 115 decibels. What intensity I is needed to produce this level?

(c) Compute $L(1)$. How does this compare with the threshold of pain which is around 140 decibels?

75. The pH of a solution is a measure of its acidity or alkalinity. Specifically, $\mathrm{pH} = -\log_{10}\left[\mathrm{H}^+\right]$ where $\left[\mathrm{H}^+\right]$ is the hydrogen ion concentration in moles per liter. A solution with a pH less than 7 is an acid, one with a pH greater than 7 is a base (alkaline) and a pH of 7 is regarded as neutral.

(a) The hygrogen ion concentration of pure water is $\left[\mathrm{H}^+\right] = 10^{-7}$. Find its pH.

(b) Find the pH of a solution with $\left[\mathrm{H}^+\right] = 6.3\times 10^{-13}$.

(c) The pH of gastric acid (the acid in your stomach) is about 0.7. What is the corresponding hydrogen ion concentration?

4.2 Exercises

1. What is the purpose of the change of base formula? Why is it useful when using a calculator?

2. When does an extraneous solution occur? How can an extraneous solution be recognized?

In Exercises 3 – 6, use the change of base property to convert the given expression to an expression with the indicated base.

3. $\log_7 15$ to base e

4. $\log_{14} 55.875$ to base 10

5. $\log_3(x+2)$ to base 10

6. $\log(x^2+1)$ to base e

In Exercises 7 – 12, use the change of base property to approximate the logarithm to five decimal places.

7. $\log_3 12$

8. $\log_5 80$

9. $\log_6 72$

10. $\log_4\left(\frac{1}{10}\right)$

11. $\log_{\frac{3}{5}} 1000$

12. $\log_{\frac{2}{3}} 50$

In Exercises 13 – 30, solve the equation analytically.

13. $3^{2x}=5$

14. $5^{-x}=2$

15. $5^x=-2$

16. $3^{x-1}=29$

17. $9^{x-10}=1$

18. $1.005^{12x}=3$

19. $e^{-5730k}=\frac{1}{2}$

20. $3^{x-1}=2^x$

21. $2^{x+1}=5^{2x-1}$

22. $3^{2x+1}=7^{x-2}$

23. $3^{x-1}=\left(\frac{1}{2}\right)^{x+5}$

24. $7^{3+7x}=3^{4-2x}$

25. $\log_4(4x)-\log_4\left(\frac{x}{4}\right)=3$

26. $\log_4(x)-\log_4(3)=1$

27. $\log_5(x-4)+\log_5 x=1$

28. $\log_2(x-1)+\log_2(x-3)=3$

29. $\log_3(x-4)+\log_3(x+4)=2$

30. $\log_5(2x+1)+\log_5(x+2)=1$

In Exercises 31 – 45, expand the logarithm. Assume when necessary that all logarithmic quantities are defined.

31. $\ln(x^3y^2)$

32. $\log_2\left(\frac{128}{x^2+4}\right)$

33. $\log_5\left(\frac{z}{25}\right)^3$

34. $\log\left(1.23\times10^{37}\right)$ 35. $\ln\left(\frac{\sqrt{z}}{xy}\right)$ 36. $\log_c\left(\frac{x^2}{y^3}\right)$

37. $\log_{\sqrt{2}}\left(4x^3\right)$ 38. $\log_8\left(x^2\sqrt{x-3}\right)$ 39. $\log\left(1000x^3y^5\right)$

40. $\log_3\left(\frac{x^2}{81y^4}\right)$ 41. $\ln\sqrt[4]{\frac{xy}{ez}}$ 42. $\log_6\left(\frac{216}{x^3y}\right)^4$

43. $\log\left(\frac{100x\sqrt{y}}{\sqrt[3]{10}}\right)$ 44. $\log_{\frac{1}{2}}\left(\frac{4\sqrt[3]{x^2}}{y\sqrt{z}}\right)$ 45. $\ln\left(\frac{\sqrt[3]{x}}{10\sqrt{yz}}\right)$

In Exercises 46 – 48, expand the logarithm after factoring. Assume when necessary that all quantities represent positive real numbers.

46. $\log_5\left(x^2-25\right)$ 47. $\ln\left(\frac{x^3\left(x^2-4\right)}{\sqrt{x^2+4}}\right)$ 48. $\log_{\frac{1}{3}}\left(9x\left(y^3-8\right)\right)$

In Exercises 49 – 60, use the properties of logarithms to write the expression as a single logarithm.

49. $\log\left(2x^4\right)+\log\left(3x^5\right)$ 50. $\ln\left(6x^9\right)-\ln\left(3x^2\right)$ 51. $4\ln x+2\ln y$

52. $\log_2 x+\log_2 y+\log_2 z$ 53. $\log_3 x-2\log_3 y$ 54. $\frac{1}{2}\log_3 x-2\log_3 y-\log_3 z$

55. $2\ln x-3\ln y-4\ln z$ 56. $\log x-\frac{1}{3}\log z+\frac{1}{2}\log y$ 57. $-\frac{1}{3}\ln x-\frac{1}{3}\ln y+\frac{1}{3}\ln z$

58. $\log_5 x-3$ 59. $3-\log x$ 60. $\log_7 x+\log_7\left(x-3\right)-2$

61. Use the given values $\ln a=2$, $\ln b=3$ and $\ln c=5$ to evaluate the following expressions.

(a) $\ln\left(\frac{a^2}{b^3c^4}\right)$

(b) $\ln\sqrt{a^2b^3c^4}$

62. Provide specific values for x, y and b to show that, in general,

(a) $\log_b\left(x+y\right)\neq\log_b x+\log_b y$

(b) $\log_b\left(x-y\right)\neq\log_b x-\log_b y$

(c) $\log_b\left(\dfrac{x}{y}\right) \neq \dfrac{\log_b x}{\log_b y}$

63. Research the history of logarithms, including the origin of the word ‘logarithm’ itself. Why is the abbreviation of the natural logarithm ‘ln’ and not ‘nl’?

64. There is a scene in the movie ‘Apollo 13’ in which several people at Mission Control use slide rules to verify a computation. Was that scene accurate? Look for other pop culture references to logarithms and slide rules.

4.3 Exercises

1. Give an example of a type of exponential equation that cannot be solved using the strategy at the beginning of this section.

2. How many intercepts will the graph of an exponential function of the form $f(x)=a^{bx+c}$, $a>0$, $a\neq 1$, have?

In Exercises 3 – 35, solve the equation analytically.

3. $64\cdot 4^{3x}=16$
4. $3^{2x+1}\cdot 3^{x}=243$
5. $2^{-3n}\cdot\frac{1}{4}=2^{n+2}$
6. $625\cdot 5^{3x+3}=125$
7. $2e^{6x}=13$
8. $e^{r+10}-10=-42$
9. $2000e^{0.1t}=4000$
10. $-8\cdot 10^{p+7}=-24$
11. $7e^{3n-5}+5=-89$
12. $e^{-3k}+6=44$
13. $-5e^{9x-8}-8=-62$
14. $-6e^{9x+8}+2=-74$
15. $7e^{8x+8}-5=-95$
16. $4e^{3x+3}-7=53$
17. $8e^{-5x-2}-4=-90$
18. $10e^{8x+3}+2=8$
19. $3e^{3-3x}+6=-31$
20. $500\left(1-e^{2x}\right)=250$
21. $30-6e^{-0.1x}=20$
22. $\dfrac{100e^{x}}{e^{x}+2}=50$
23. $\dfrac{5000}{1+2e^{-3t}}=2500$
24. $\dfrac{150}{1+29e^{-0.8t}}=75$
25. $25\left(\dfrac{4}{5}\right)^{x}=10$
26. $e^{2x}=2e^{x}$
27. $7e^{2x}=28e^{-6x}$
28. $e^{2x}-e^{x}-132=0$
29. $e^{2x}-e^{x}-6=0$
30. $e^{2x}-3e^{x}-10=0$
31. $e^{2x}=e^{x}+6$
32. $4^{x}+2^{x}=12$
33. $e^{x}-3e^{-x}=2$
34. $e^{x}+15e^{-x}=8$
35. $3^{x}+25\cdot 3^{-x}=10$

In Exercises 36 – 50, sketch the graph of $y=f(x)$. State the domain, range, *x*- and *y*-intercepts, and equation of the asymptote. Draw asymptotes as dashed lines on your graph.

36. $f(x)=3^{\frac{x}{2}}-2$
37. $f(x)=\left(\dfrac{1}{3}\right)^{-x}-1$
38. $f(x)=4^{-x+1}$
39. $f(x)=5^{x-1}-2$
40. $f(x)=-5^{x+2}+3$
41. $f(x)=-\left(\dfrac{1}{2}\right)^{x}-3$

42. $f(x) = e^{-x} + 2$

43. $f(x) = 2 - e^{x}$

44. $f(x) = 8 - e^{-x}$

45. $f(x) = 3(2^{x}) + 1$

46. $f(x) = 5(3^{-x})$

47. $f(x) = 2\left(\frac{1}{3}\right)^{-x}$

48. $f(x) = 2^{x^2}$

49. $f(x) = 2^{1-x^2}$

50. $f(x) = 8 - 2^{x^2}$

In Exercises 51 – 53, determine the solution to $f(x) > 0$ for the given function. Hint: Start by finding the *x*-intercept and graphing the function over its domain.

51. $f(x) = 3^{\frac{x}{2}} - 1$

52. $f(x) = 3^{-x+2}$

53. $f(x) = 3 - e^{x}$

In Exercises 54 – 56, find the inverse of the given function.

54. $f(x) = 4^{x+5}$

55. $f(x) = e^{2x+5} + 5$

56. $f(x) = b^{-3x+7} + 4$

57. The population of a small town is modeled by the equation $P = 1650e^{0.5t}$ where t is measured in years. In approximately how many years will the town's population reach 20,000?

58. Atmospheric pressure P in pounds per square inch is represented by the formula $P = 14.7e^{-0.21x}$, where *x* is the number of miles above sea level. To the nearest foot, how high is the peak of a mountain with an atmospheric pressure of 8.369 pounds per square inch?

4.4 Exercises

1. What type(s) of transformation(s), if any, affect the domain of a logarithmic function?

2. What type(s) of transformation(s), if any, affect the range of a logarithmic function?

In Exercises 3 – 14, find the domain of the function.

3. $f(x)=\ln(2-x)$

4. $f(x)=\log\left(x-\frac{3}{7}\right)$

5. $h(x)=-\log(3x-4)+3$

6. $g(x)=\ln(2x+6)-5$

7. $f(x)=\log_3(15-5x)+6$

8. $f(x)=\ln(x^2+4)$

9. $f(x)=\log_7(4x+8)$

10. $f(x)=\ln(4x-20)$

11. $f(x)=\log(x^2+9x+18)$

12. $f(x)=\log\left(\frac{x+2}{x^2-1}\right)$

13. $f(x)=\log\left(\frac{x^2+9x+18}{4x-20}\right)$

14. $f(x)=\ln(7-x)+\ln(x-4)$

In Exercises 15 – 32, solve the equation analytically.

15. $\log(3x-1)=\log(4-x)$

16. $\log_2 x^3=\log_2 x$

17. $\ln(8-x^2)=\ln(2-x)$

18. $\log_5(18-x^2)=\log_5(6-x)$

19. $\ln(x^2-99)=0$

20. $\log\left(\frac{x}{10^{-3}}\right)=4.7$

21. $-\log x=5.4$

22. $10\log\left(\frac{x}{10^{-12}}\right)=150$

23. $6-3\log_5(2x)=0$

24. $\ln(x-2)=1+\ln x$

25. $\log_{169}(3x+7)-\log_{169}(5x-9)=\frac{1}{2}$

26. $\ln(x+1)-\ln x=3$

27. $2\log_7 x=\log_7 2+\log_7(x+12)$

28. $\log x-\log 2=\log(x+8)-\log(x+2)$

29. $\log_2 x=\log_4(3x+28)$

30. $\log_3(x-2)=\log_{27}(4x+27)$

31. $\log_3 x + \log_{243}\left(x^5\right) + 3 = 0$

32. $\log_3 x = \log_{\frac{1}{3}} x + 8$

In Exercises 33 – 44, sketch the graph of $y = f(x)$. State the domain, range, *x*- and *y*-intercepts, and equation of the asymptote. Draw asymptotes as dashed lines on your graph.

33. $f(x) = \log(x+2) - 1$

34. $f(x) = -\ln(8-x)$

35. $f(x) = -10\ln\left(\frac{x}{10}\right)$

36. $f(x) = \log_2(x+1)$

37. $f(x) = \log_3(-x)$

38. $f(x) = \log_2(-x+3)$

39. $f(x) = -\log_3(x-2) - 4$

40. $f(x) = \ln(x-1)$

41. $f(x) = \log(x-3) + 2$

42. $f(x) = \log\left(\frac{1}{2}x - 1\right)$

43. $f(x) = \log_2|x|$

44. $f(x) = \log_2|x+1|$

In Exercises 45 – 47, determine the solution to $f(x) > 0$ for the given function. Hint: Start by finding the *x*-intercept and graphing the function over its domain.

45. $f(x) = \log(x+5) + 3$

46. $f(x) = \ln(-x+4)$

47. $f(x) = -\log_4(x+2) - 7$

In Exercises 48 – 50, find the inverse of the given function.

48. $f(x) = \log_2(x-11)$

49. $f(x) = -\ln(5-x)$

50. $f(x) = -\log_7(x-3) + 10$

51. Let $P(t)$ be the population of Arizona, in millions, *t* years after 1970. In 1970 Arizona had a population of 1.8 million. Assuming a constant growth rate of 3%, the population of Arizona satisfies the equation $\ln P(t) = \ln 1.8 + 0.03t$. Use this equation to estimate the population of Arizona in 2018. The population of Arizona in 2018 is about 7.34 million. Compare this with your estimate.

52. Let $P(t)$ be the population of the United States of America, in millions, *t* years after 1970. In 1970, the USA had a population of about 210 million. Assuming a constant growth rate of 1%, the population of the USA satisfies the equation $\ln P(t) = \ln 210 + 0.01t$. Use this equation to estimate the population of the USA in 2018. The population of the USA in 2018 is about 327 million. Compare this with your estimate.

53. Let $P(t)$ be the population of Canada, in millions, *t* years after 1970. In 1970, Canada had a population of about 21.5 million. Assuming a constant growth rate of 1.2%, the population of Canada satisfies the equation $\ln P(t) = \ln 21.5 + 0.012t$. Use this equation to estimate the population of

Canada in 2018. The population of Canada in 2018 is about 36.9 million. Compare this with your estimate.

54. Let $P(t)$ be the population of China, in millions, t years after 1970. In 1970, China had a population of about 825 million people. Assuming a constant growth rate of 0.5%, the population of China satisfies the equation $\ln P(t) = \ln 825 + 0.005t$. Use this equation to estimate the population of China in 2018. The population of China in 2018 is about 1415 million. Compare this with your estimate.

4.5 Exercises

1. What is the effect of interest on a savings account being compounded monthly versus quarterly?

2. How is continuously compounded interest related to exponential growth and decay?

In Exercises 3 – 8, find each of the following.

(a) the amount A in the account as a function of the term of the investment t in years;

(b) how much is in the account after 5 years, 10 years, 30 years and 35 years, rounding your answers to the nearest cent;

(c) how long it will take the initial investment to double, rounding your answer to the nearest year.

3. \$500 is invested in an account that offers 0.75%, compounded monthly.

4. \$500 is invested in an account that offers 0.75%, compounded continuously.

5. \$1000 is invested in an account that offers 1.25%, compounded monthly.

6. \$1000 is invested in an account that offers 1.25%, compounded continuously.

7. \$5000 is invested in an account that offers 2.125%, compounded monthly.

8. \$5000 is invested in an account that offers 2.125%, compounded continuously.

9. Look back at your answers to Exercises 3 – 8. What can be said about the difference between monthly compounding and continuously compounding the interest in those situations? With the help of your classmates, discuss scenarios where the difference between monthly and continuously compounded interest would be more dramatic. Try varying the interest rate, the term of the investment and the principal. Use computations to support your answer.

10. How much money needs to be invested now to obtain \$2000 in 3 years if the interest rate in a savings account is 0.25%, compounded continuously? Round your answer to the nearest cent.

11. How much money needs to be invested now to obtain \$5000 in 10 years if the interest rate in a CD is 2.25%, compounded monthly? Round your answer to the nearest cent.

12. If the Annual Percentage Rate (APR) for a savings account is 0.25% compounded monthly, use the equation $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ to answer the following.

    (a) For a principal of \$2000, how much is in the account after 8 years?

(b) If the original principal was \$2000 and the account now contains \$4000, how many years have passed since the original investment, assuming no other additions or withdrawals have been made?

(c) What principal should be invested so that the account balance is \$2000 in three years?

13. If the Annual Percentage Rate (APR) for a 36-month Certificate of Deposit (CD) is 2.25%, compounded monthly, use the equation $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ to answer the following.

(a) For a principal of \$2000, how much is in the account after 8 years?

(b) If the original principal was \$2000 and the account now contains \$4000, how many years have passed since the original investment, assuming no other additions or withdrawals have been made?

(c) What principal should be invested so that the account balance is \$2000 in three years?

(d) The Annual Percentage Yield is the simple[16] interest rate that returns the same amount of interest after one year as the compound interest does. Compute the APY for this investment.

14. A finance company offers a promotion on \$5000 loans. The borrower does not have to make any payments for the first three years, however interest will continue to be charged to the loan at 29.9% compounded continuously. What amount will be due at the end of the three year period, assuming no payments are made? If the promotion is extended an additional three years, and no payments are made, what amount will be due?

15. Use the equation $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ to show that the time it takes for an investment to double in value does not depend on the principal *P*, but rather depends on the APR and the number of compoundings per year. Let $n = 12$ and with the help of your classmates compute the doubling time for a variety of rates *r*. Then look up the Rule of 72 and compare your answers to what that rule says. If you're really interested[17] in Financial Mathematics, you could also compare and contrast the Rule of 72 with the Rule of 70 and the Rule of 69.

[16] There is no compounding with simple interest.
[17] Awesome pun!

In Exercises 16 – 20, we list some radioactive isotopes and their associated half-lives. Assume that each decays according to the formula $A(t) = A_0 e^{kt}$ where A_0 is the initial amount of the material and *k* is a constant representing the rate of decay. For each isotope:

(a) Find the decay constant *k*. Round your answer to four decimal places.

(b) Find a function that gives the amount of isotope *A* that remains after time *t*. (Keep the units of *A* and *t* the same as the given data.

(c) Determine how long it takes for 90% of the material to decay. Round your answer to two decimal places. (HINT: If 90% of the material decays, how much is left?)

16. Cobalt-60, used in food irradiation, initial amount 50 grams, half-life of 5.27 years.

17. Phosphorus-32, used in agriculture, initial amount 2 milligrams, half-life 14 days.

18. Chromium-51, used to track red blood cells, initial amount 75 milligrams, half-life 27.7 days.

19. Americium-241, used in smoke detectors, initial amount 0.29 micrograms, half-life 432.7 years.

20. Uranium-235, used for nuclear power, initial amount 1 kg, half-life 704 million years.

21. With the help of your classmates, show that the time it takes for 90% of each isotope listed in Exercises 16 – 20 to decay does not depend on the initial amount of the substance, but rather on only the decay constant *k*. Find a formula, in terms of *k* only, to determine how long it takes for 90% of a radioactive isotope to decay.

22. The Gross Domestic Product (GDP) of the US (in billions of dollars) *t* years after the year 2000 can be modeled by

$$G(t) = 9743.77e^{0.0514t}$$

(a) Find and interpret $G(0)$.

(b) According to the model, what should have been the GDP in 2007? In 2010? (According to the US Department of Commerce, the 2007 GDP was $14,369.1 billion and the 2010 GDP was $14,657.8 billion.)

23. The diameter *D* of a tumor, in millimeters, *t* days after it is detected is given by

$$D(t) = 15e^{0.0277t}$$

(a) What was the diameter of the tumor when it was originally detected?

(b) How long until the diameter of the tumor doubles?

24. Under optimal conditions, the growth of a certain strain of *E. coli* is modeled by the Law of Uninhibited Growth $A(t) = A_0 e^{kt}$ where A_0 is the initial number of bacteria and t is the elapsed time, measured in minutes. From numerous experiments, it has been determined that the doubling time of this organism is 20 minutes. Suppose 1000 bacteria are present initially.

    (a) Find the growth constant k. Round your answer to four decimal places.

    (b) Find a function that gives the number of bacteria $A(t)$ after t minutes.

    (c) How long until there are 9000 bacteria? Round your answer to the nearest minute.

25. Yeast is often used in biological experiments. A research technician estimates that a sample of yeast suspension contains 2.5 million organisms per cubic centimeter (cc). Two hours later, she estimates the population density to be 6 million organisms per cc. Let t be the time elapsed since the first observation, measured in hours. Assume that the yeast growth follows the Law of Uninhibited Growth $A(t) = A_0 e^{kt}$.

    (a) Find the growth constant k. Round your answer to four decimal places.

    (b) Find a function that gives the number of yeast (in millions) per cc $A(t)$ after t hours.

    (c) What is the doubling time for this strain of yeast?

26. The Law of Uninhibited Growth also applies to situations where an animal is re-introduced into a suitable environment. Such a case is the reintroduction of wolves to Yellowstone National Park. According to the National Park Service, the wolf population in Yellowstone National Park was 52 in 1996 and 118 in 1999. Using these data, find a function of the form $A(t) = A_0 e^{kt}$ that models the number of wolves t years after 1996. (Use $t = 0$ to represent the year 1996. Also, round your value of k to four decimal places.) According to the model, how many wolves were in Yellowstone in 2002? (The recorded number is 272.)

27. During the early years of a community, it is not uncommon for the population to grow according to the Law of Uninhibited Growth. According to the Painesville Wikipedia entry, in 1860, the village of Painesville had a population of 2649. In 1920, the population was 7272. Use these two data points to fit a model of the form $A(t) = A_0 e^{kt}$ where $A(t)$ is the number of Painesville Residents t years after 1860. (Use $t = 0$ to represent the year 1860. Also, round the value of k to four decimal places.) According to this model, what was the population of Painesville in 2010? (The 2010 census gave the population as 19,563.) What could be some causes for such a vast discrepancy?

28. The population of Sasquatch in Salt Lake County is modeled by

$$P(t) = \frac{120}{1 + 3.167e^{-0.05t}}$$

where $P(t)$ is the population of Sasquatch t years after 2010.

(a) Find and interpret $P(0)$.

(b) Find the population of Sasquatch in Salt Lake County in 2013. Round your answer to the nearest Sasquatch.

(c) When will the population of Sasquatch in Salt Lake County reach 60? Round your answer to the nearest year.

29. The half-life of the radioactive isotope Carbon-14 is about 5730 years.

(a) Use the equation $A(t) = A_0 e^{kt}$ to express the amount of Carbon-14 left from an initial N milligrams as a function of time t in years.

(b) What percentage of the original amount of Carbon-14 is left after 20,000 years?

(c) If an old wooden tool is found in a cave and the amount of Carbon-14 present in it is estimated to be only 42% of the original amount, approximately how old is the tool?

(d) Radiocarbon dating is not as easy as these exercises might lead you to believe. With the help of your classmates, research radiocarbon dating and discuss why our model is somewhat over-simplified.

30. Carbon-14 cannot be used to date inorganic material such as rocks, but there are many other methods of radiometric dating which estimate the age of rocks. One of them, Rubidium-Strontium dating, uses Rubidium-87 which decays to Strontium-87 with a half-life of 50 billion years. Use the equation $A(t) = A_0 e^{kt}$ to express the amount of Rubidium-87 left from an initial 2.3 micrograms as a function of time t in billions of years. Research this and other radiometric techniques and discuss the margins of error for various methods with your classmates.

31. Use the equation $A(t) = A_0 e^{kt}$ to show that $k = -\frac{\ln 2}{h}$ where h is the half-life of the radioactive isotope.