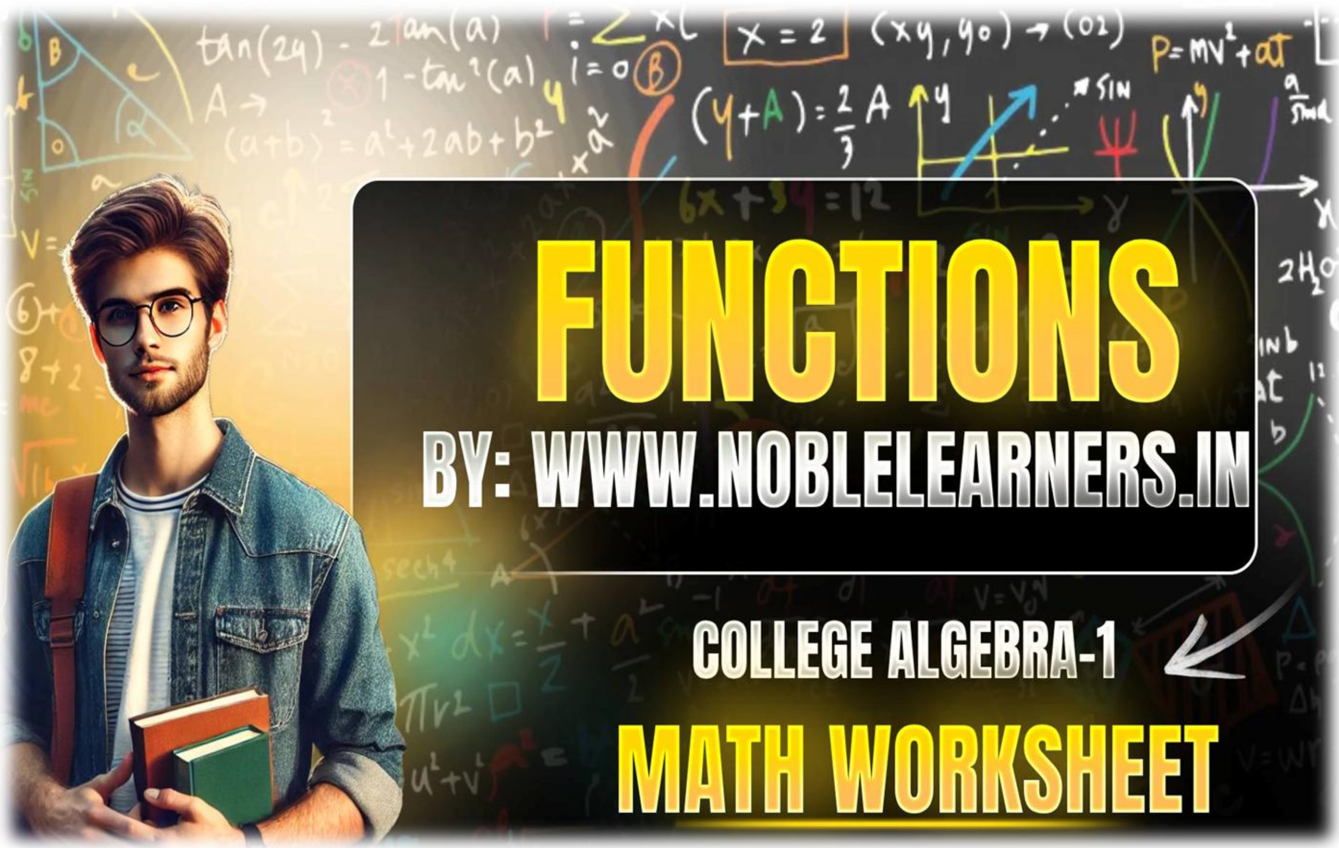

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1.1 Exercises

1. What is the difference between a relation and a function?

2. Why does the vertical line test tell us whether the graph of a relation represents a function?

In Exercises 3 – 8, determine whether or not the relation represents a function. Find the domain and range of those relations that are functions.

3. $\{(-3,9), (-2,4), (-1,1), (0,0), (1,1), (2,4), (3,9)\}$

4. $\{(-3,0), (1,6), (2,-3), (4,2), (-5,6), (4,-9), (6,2)\}$

5. $\{(-3,0), (-7,6), (5,5), (6,4), (4,9), (3,0)\}$

6.

x	y
−5	−4
1	2
4	5
8	7
11	16

7.

x	y
0	−2
3	2
3	6
8	7
12	16

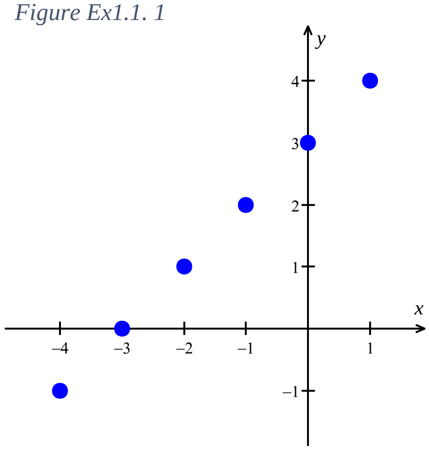
8.

x	y
−5	−1
1	3
6	3
8	9
16	10

In Exercises 9 – 30, determine if the relation represents y as a function of x. Find the domain and range of those relations that are functions.

9.

Figure Ex1.1. 1



10.

Figure Ex1.1. 2

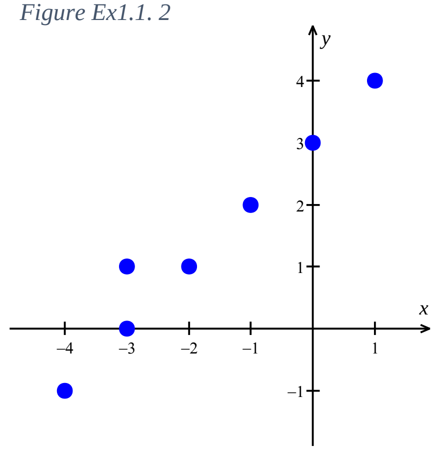


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Figure Ex1.1. 3

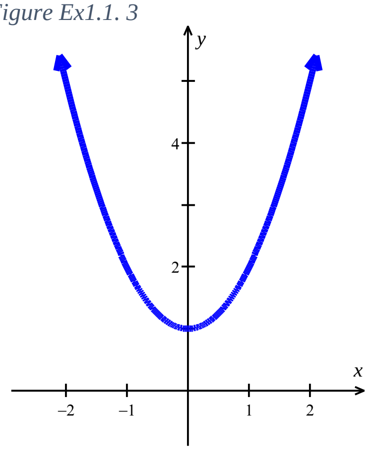

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Figure Ex1.1. 4

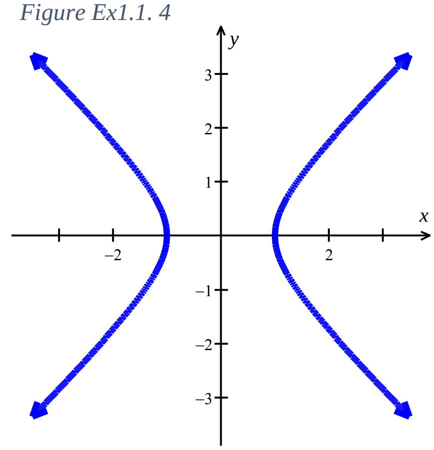

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Figure Ex1.1. 5

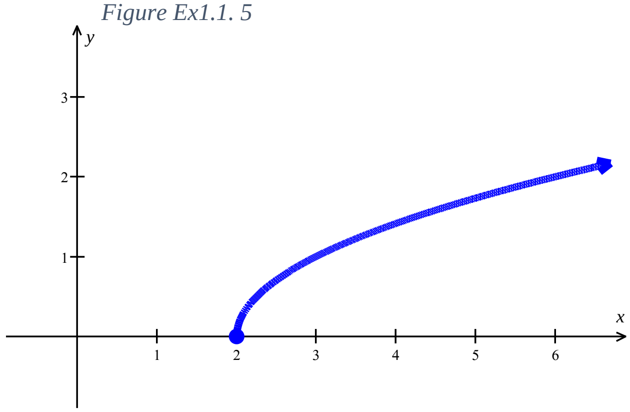

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Figure Ex1.1. 6

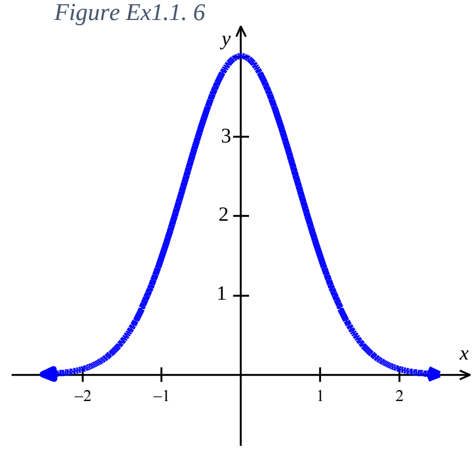

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Figure Ex1.1. 7

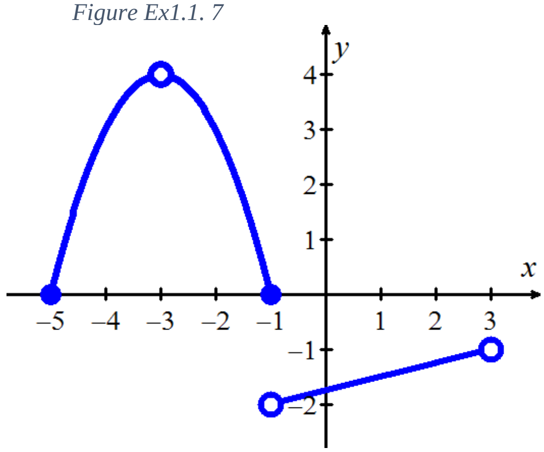

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Figure Ex1.1. 8

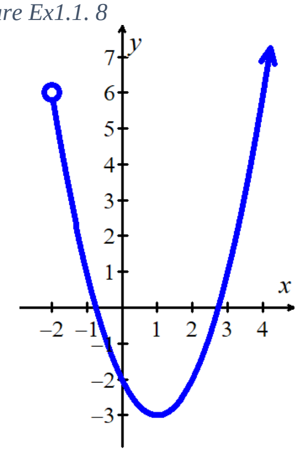

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Figure Ex1.1. 9

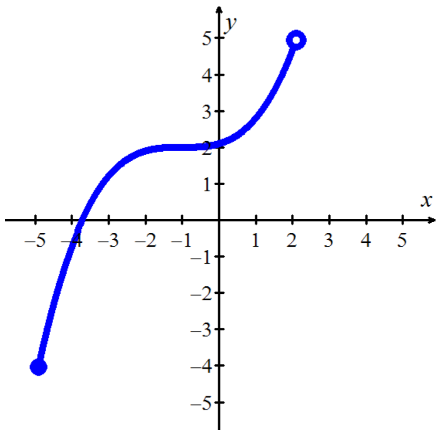

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Figure Ex1.1. 10

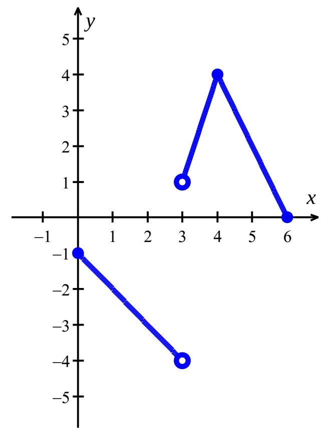

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Figure Ex1.1. 11

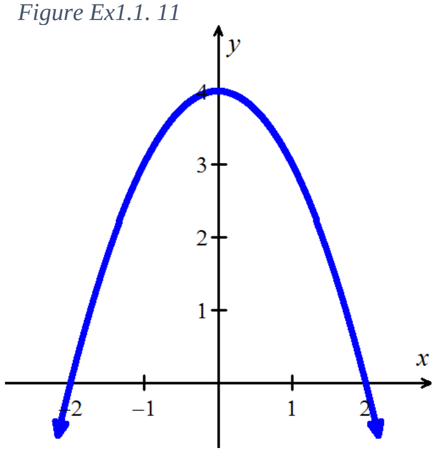

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Figure Ex1.1. 12

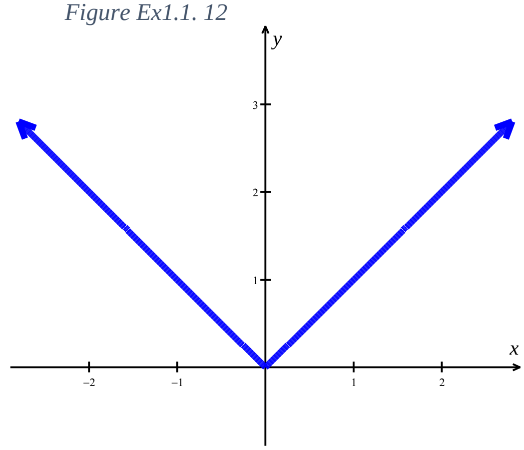

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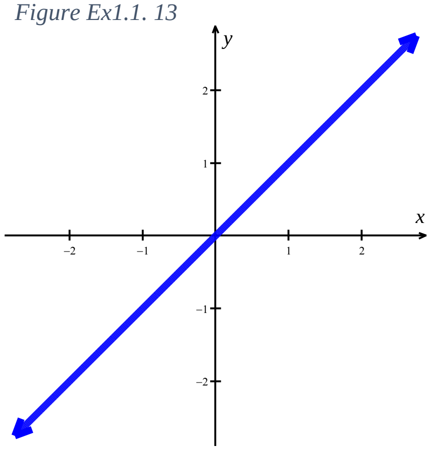
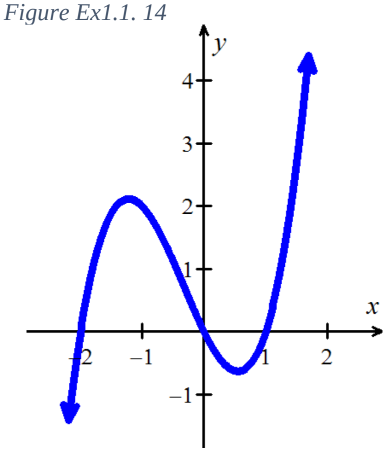
Figure Ex1.1. 13



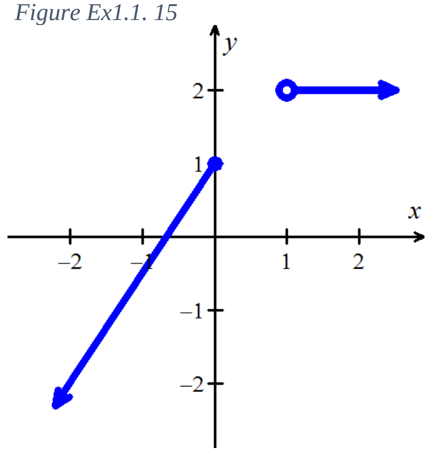
22.

Figure Ex1.1. 14



23.

Figure Ex1.1. 15



24.

Figure Ex1.1. 16

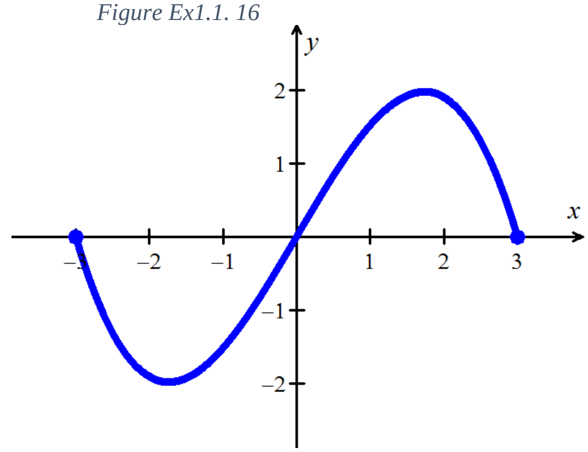


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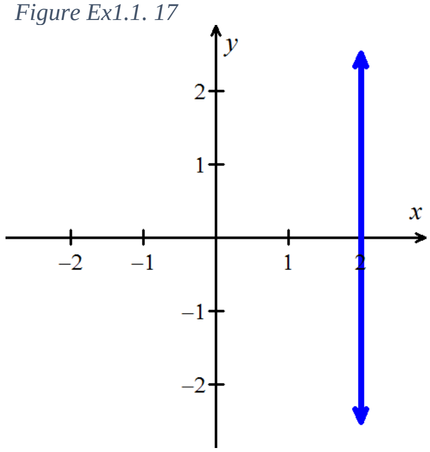
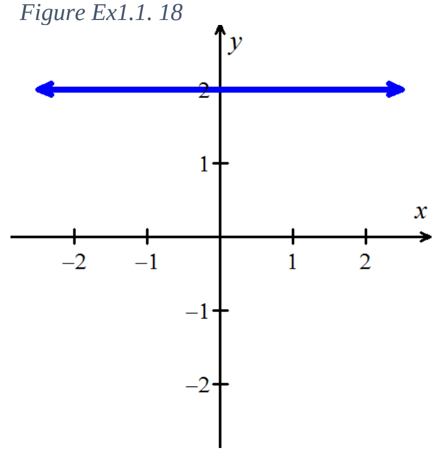
Figure Ex1.1. 17



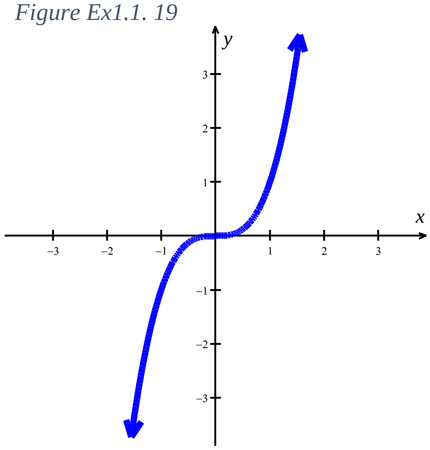
26.

Figure Ex1.1. 18



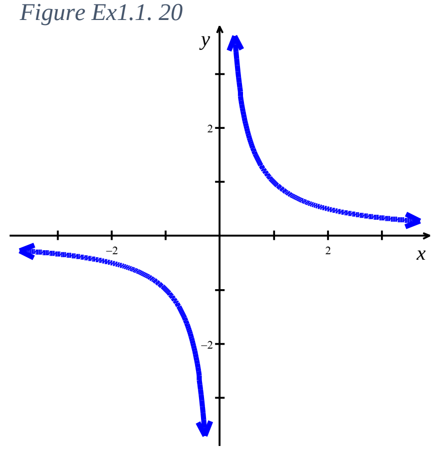
27.

Figure Ex1.1. 19



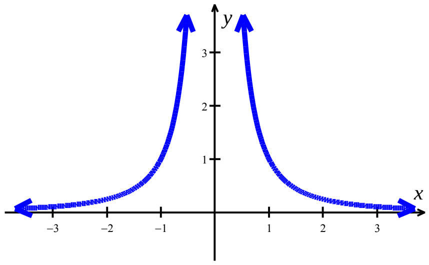
28.

Figure Ex1.1. 20



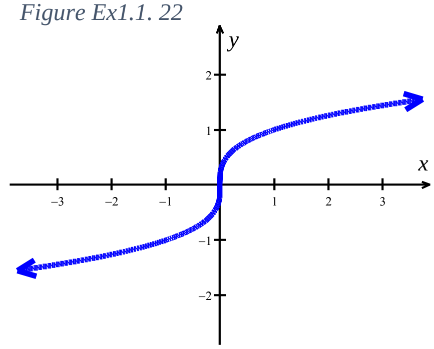
29.

Figure Ex1.1. 21



30.

Figure Ex1.1. 22



In Exercises 31 – 45, determine if the relation, expressed as an equation, is a function. Assume that x is the independent variable and y is the dependent variable. Rewrite the equation as necessary to verify your conclusion that y is, or is not, a function of x.

31. $y = x^3 - x$

32. $y = \sqrt{x-2}$

33. $x^3 y = -4$

34. $x^2 - y^2 = 1$

35. $y = \dfrac{x}{x^2 - 9}$

36. $x = -6$

37. $x = y^2 + 4$

38. $y = x^2 + 4$

39. $x^2 + y^2 = 4$

40. $y = \sqrt{4 - x^2}$

41. $x^2 - y^2 = 4$

42. $x^3 + y^3 = 4$

43. $2x + 3y = 4$

44. $2xy = 4$

45. $x^2 = y^2$

In Exercises 46 – 55, use the given function f to evaluate, if possible, and simplify the following:

(a) $f(3)$ (b) $f(-1)$ (c) $f\left(\dfrac{3}{2}\right)$ (d) $f(a)$

46. $f(x) = 2x + 1$

47. $f(x) = 3 - 4x$

48. $f(x) = 2 - x^2$

49. $f(x) = x^2 - 3x + 2$

50. $f(x) = \dfrac{x}{x-1}$

51. $f(x) = \dfrac{2}{x^3}$

52. $f(x) = \sqrt{x-2}$

53. $f(x) = \sqrt[3]{x}$

54. $f(x) = 6$

55. $f(x) = 0$

In Exercises 56 – 63, use the given function f to evaluate, if possible, and simplify the following:

(a) $f(2)$ (b) $f(-2)$ (c) $f(h)$ (d) $f(0)$

56. $f(x)=2x-5$
57. $f(x)=5-2x$
58. $f(x)=2x^2-1$
59. $f(x)=3x^2+3x-2$
60. $f(x)=\sqrt{2x+1}$
61. $f(x)=117$
62. $f(x)=\dfrac{x}{2}$
63. $f(x)=\dfrac{2}{x}$

In Exercises 64 – 67, use the given function f to solve $f(x)=4$.

64. $f(x)=2x-8$
65. $f(x)=7-3x$
66. $f(x)=2x^2-14$
67. $f(x)=x^2+x+2$

In Exercises 68 – 75, use the given function f to find $f(0)$ and to solve $f(x)=0$, if possible. Then use your answers to write the resulting ordered pairs.[8]

68. $f(x)=2x-1$
69. $f(x)=3-\dfrac{2}{5}x$
70. $f(x)=2x^2-6$
71. $f(x)=x^2-x-12$
72. $f(x)=\sqrt{x+4}$
73. $f(x)=\sqrt{1-2x}$
74. $f(x)=\dfrac{3}{4-x}$
75. $f(x)=\dfrac{3x}{4-x^2}$

In Exercises 76 – 93, find the domain of the function.

76. $f(x)=x^4-13x^3+56x^2-19$
77. $f(x)=x^4+4$
78. $f(x)=\dfrac{x-2}{x+1}$
79. $f(x)=\dfrac{3x}{x^2+x-2}$

[8] Note that you have found points on the *x*- and *y*-axes. These are referred to as *x*- and *y*-intercepts, respectively. We will talk more about this soon.

80. $f(x)=\dfrac{2x}{x^2+4}$

81. $f(x)=\dfrac{2x}{x^2-4}$

82. $f(x)=\dfrac{x+4}{x^2-36}$

83. $f(x)=\dfrac{x-2}{x-2}$

84. $f(x)=\sqrt{3-x}$

85. $f(x)=\sqrt{2x+5}$

86. $f(x)=\sqrt{x+3}$

87. $f(x)=\dfrac{\sqrt{7-x}}{x^2+1}$

88. $f(x)=\sqrt{6x-2}$

89. $f(x)=\dfrac{6}{\sqrt{6x-2}}$

90. $f(x)=\sqrt[3]{6x-2}$

91. $f(x)=\dfrac{\sqrt{6x-2}}{x^2-36}$

92. $s(t)=\dfrac{t}{t-8}$

93. $Q(r)=\dfrac{\sqrt{r}}{r-8}$

1.2 Exercises

1. How can you determine whether a function is odd or even from the formula for the function?

2. How are the absolute maximum and minimum similar to and different from the local extrema?

3. Compute the following function values for $f(x)=\begin{cases} x+5 & \text{if} \quad x\le -3 \\ \sqrt{9-x^2} & \text{if} \quad -3<x\le 3 \\ -x+5 & \text{if} \quad x>3 \end{cases}$

(a) $f(-4)$ (b) $f(-3)$ (c) $f(3)$

(d) $f(3.001)$ (e) $f(-3.001)$ (f) $f(2)$

4. Compute the following function values for $f(x)=\begin{cases} x^2 & \text{if} \quad x\le -1 \\ \sqrt{1-x^2} & \text{if} \quad -1<x\le 1 \\ x & \text{if} \quad x>1 \end{cases}$

(a) $f(4)$ (b) $f(-3)$ (c) $f(1)$

(d) $f(0)$ (e) $f(-1)$ (f) $f(-0.999)$

In Exercises 5 – 16, sketch the graph of the given piecewise-defined function.

5. $f(x)=\begin{cases} x+1 & \text{if } x<-2 \\ -2x-3 & \text{if } x\ge -2 \end{cases}$

6. $f(x)=\begin{cases} 2x-1 & \text{if } x<1 \\ 1+x & \text{if } x\ge 1 \end{cases}$

7. $f(x)=\begin{cases} x+1 & \text{if } x<0 \\ x-1 & \text{if } x>0 \end{cases}$

8. $f(x)=\begin{cases} 3 & \text{if } x<0 \\ \sqrt{x} & \text{if } x\ge 0 \end{cases}$

9. $f(x)=\begin{cases} x^2 & \text{if } x<0 \\ 1-x & \text{if } x>0 \end{cases}$

10. $f(x)=\begin{cases} x^2 & \text{if } x<0 \\ x+2 & \text{if } x\ge 0 \end{cases}$

11. $f(x)=\begin{cases} x+1 & \text{if } x<1 \\ x^3 & \text{if } x\ge 1 \end{cases}$

12. $f(x)=\begin{cases} |x| & \text{if } x<2 \\ 1 & \text{if } x\ge 2 \end{cases}$

13. $f(x)=\begin{cases} x^2 & \text{if } x\le 0 \\ 2x & \text{if } x>0 \end{cases}$

14. $f(x)=\begin{cases} -3 & \text{if} \quad x<0 \\ 2x-3 & \text{if} \quad 0\le x\le 3 \\ 3 & \text{if} \quad x>3 \end{cases}$

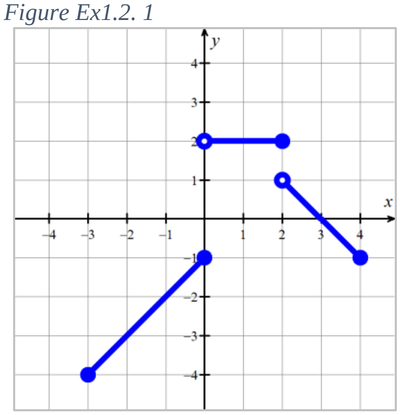
15. $f(x)=\begin{cases} x^2 & \text{if} \quad x \le -2 \\ 3-x & \text{if} \quad -2<x<2 \\ 4 & \text{if} \quad x \ge 2 \end{cases}$

16. $f(x)=\begin{cases} \dfrac{1}{x} & \text{if} \quad -6<x<-1 \\ x & \text{if} \quad -1<x<1 \\ \sqrt{x} & \text{if} \quad 1<x<9 \end{cases}$

In Exercises 17 – 22, complete the description of the piecewise-defined function.

17.

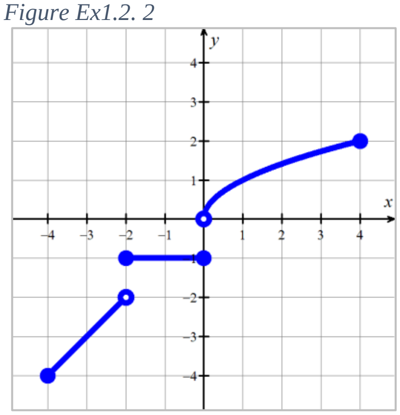
Figure Ex1.2. 1



$f(x)=\begin{cases} x-1 & \text{if} \quad \underline{\hspace{3cm}} \\ 2 & \text{if} \quad \underline{\hspace{3cm}} \\ 3-x & \text{if} \quad \underline{\hspace{3cm}} \end{cases}$

18.

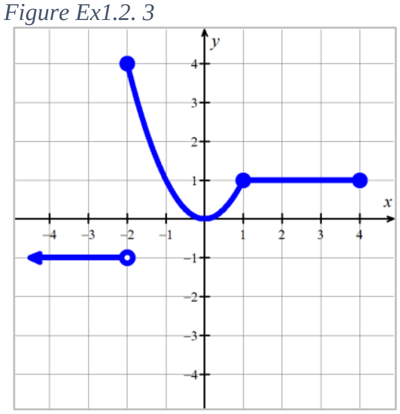
Figure Ex1.2. 2



$f(x)=\begin{cases} x & \text{if} \quad \underline{\hspace{3cm}} \\ -1 & \text{if} \quad \underline{\hspace{3cm}} \\ \sqrt{x} & \text{if} \quad \underline{\hspace{3cm}} \end{cases}$

19.

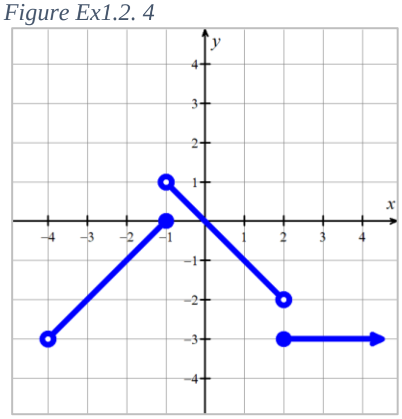
Figure Ex1.2. 3



$f(x)=\begin{cases} -1 & \text{if} \quad \underline{\hspace{3cm}} \\ x^2 & \text{if} \quad \underline{\hspace{3cm}} \\ 1 & \text{if} \quad \underline{\hspace{3cm}} \end{cases}$

20.

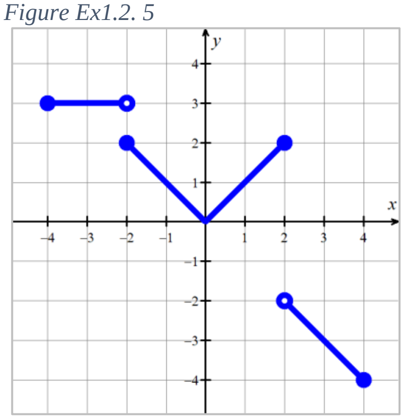
Figure Ex1.2. 4



$$f(x)=\begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -4 < x \le -1 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -1 < x < 2 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad x \ge 2 \end{cases}$$

21.

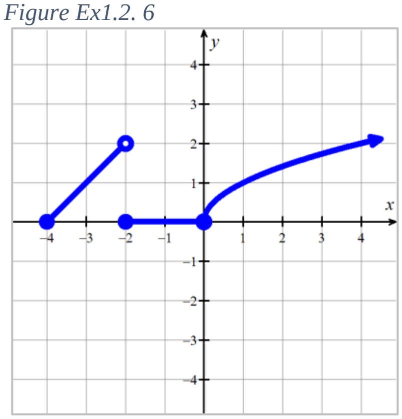
Figure Ex1.2. 5



$$f(x)=\begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -4 \le x < -2 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -2 \le x \le 2 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad 2 < x \le 4 \end{cases}$$

22.

Figure Ex1.2. 6



$$f(x)=\begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -4 \le x < -2 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad -2 \le x < 0 \\ \_\_\_\_\_\_\_\_\_\_ & \text{if} \quad x \ge 0 \end{cases}$$

In Exercises 23 – 30, does the graph appear to be symmetric about the x-axis, y-axis, origin, none of these, or all of these? For graphs representing functions, does the implied symmetry indicate that the function is even, odd, neither, or both?

23.

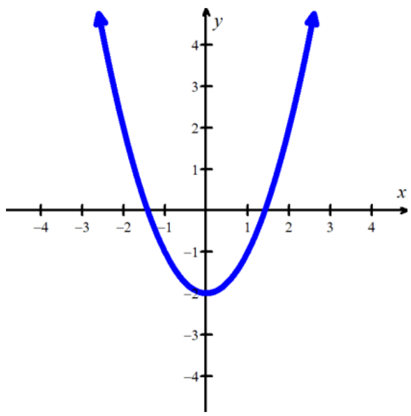


Figure Ex1.2. 7

24.

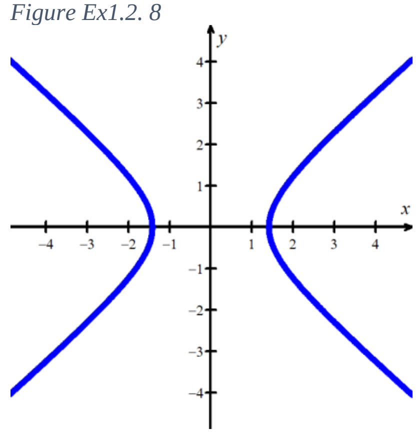


Figure Ex1.2. 8

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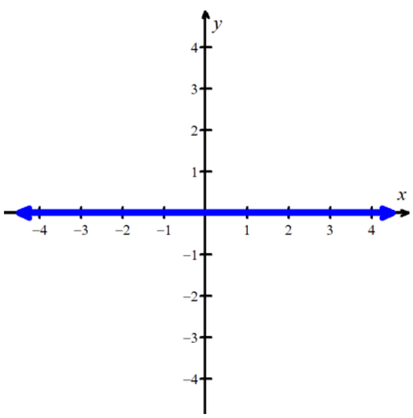


Figure Ex1.2. 9

26.

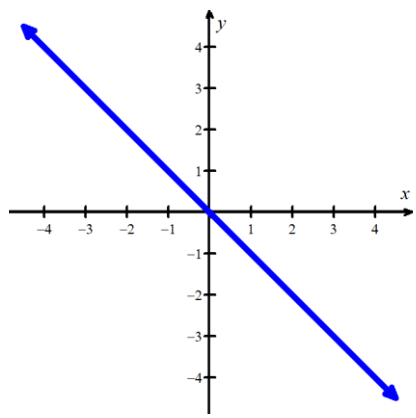


Figure Ex1.2. 10

27.

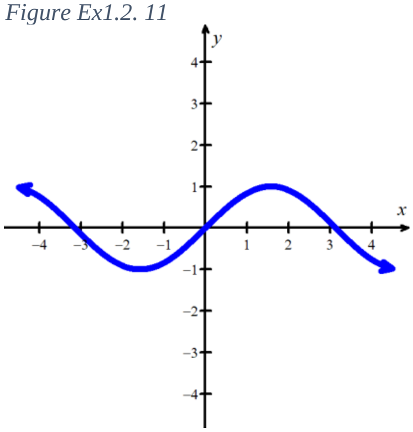


Figure Ex1.2. 11

28.

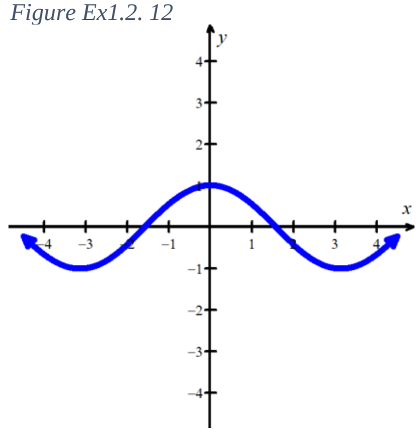
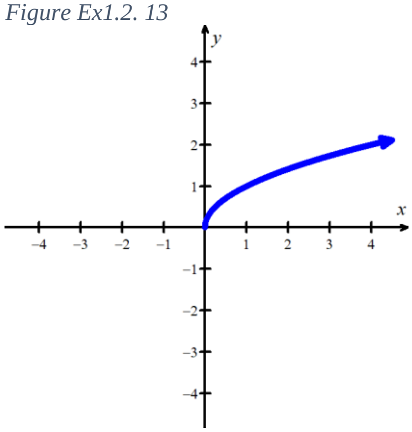


Figure Ex1.2. 12

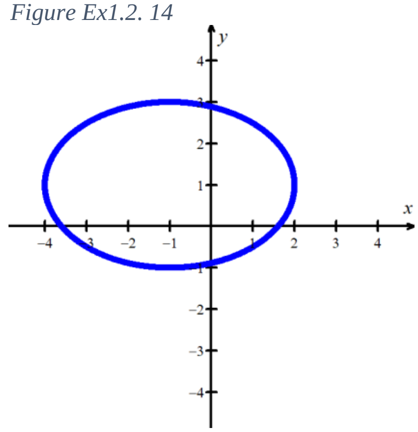
29.

Figure Ex1.2. 13



30.

Figure Ex1.2. 14



In Exercises 31 – 51, determine analytically if the function is even, odd, or neither.

31. $f(x) = 7x$

32. $f(x) = 7x + 2$

33. $f(x) = 7$

34. $f(x) = 3x^2 - 4$

35. $f(x) = -x^5 + 2x^3 - x$

36. $f(x) = x^2 - x - 6$

37. $f(x) = 2x^3 - x$

38. $f(x) = x^6 - x^4 + x^2 + 9$

39. $f(x) = 4 - x^2$

40. $f(x) = x^3 + x^2 + x + 1$

41. $f(x) = \sqrt{1 - x}$

42. $f(x) = \sqrt{1 - x^2}$

43. $f(x) = 0$

44. $f(x) = \sqrt[3]{x}$

45. $f(x) = \sqrt[3]{x^2}$

46. $f(x) = \dfrac{3}{x^2}$

47. $f(x) = \dfrac{2x - 1}{x + 1}$

48. $f(x) = \dfrac{3x}{x^2 + 1}$

49. $f(x) = \dfrac{x^2 - 3}{x - 4x^3}$

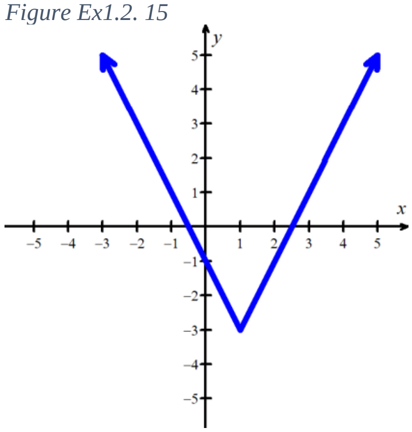
50. $f(x) = \dfrac{9}{\sqrt{4 - x^2}}$

51. $f(x) = \dfrac{\sqrt[3]{x^3 + x}}{5x}$

In Exercises 52 – 55, use the graph to estimate intervals on which the function is increasing or decreasing.

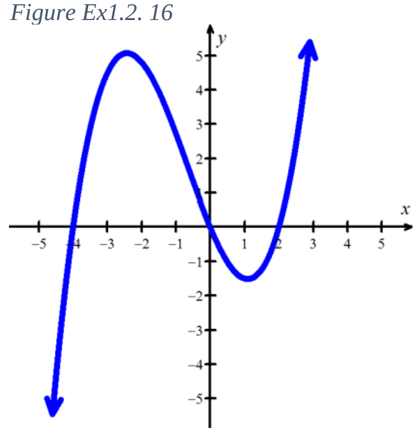
52.

Figure Ex1.2. 15



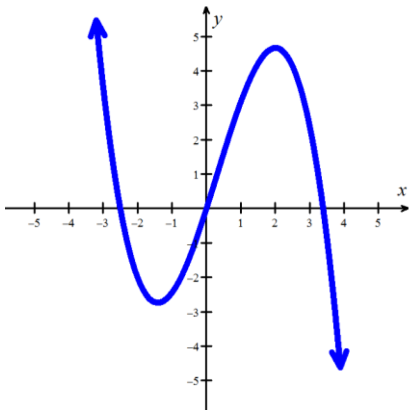
53.

Figure Ex1.2. 16



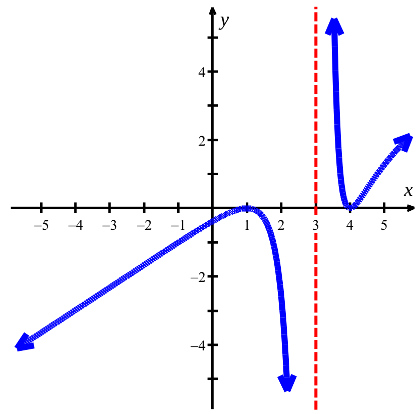
54.

Figure Ex1.2. 17



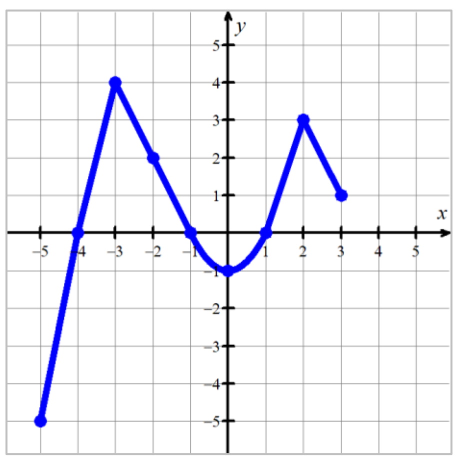
55.

Figure Ex1.2. 18



In Exercises 56 – 71, use the graph of $y = f(x)$ given below to answer the question.

Figure Ex1.2. 19



$y = f(x)$

56. Find the domain of f.

57. Find the range of f.

58. Determine $f(-2)$.

59. Solve $f(x) = 4$.

60. List the *x*-intercepts, if any exist.

61. List the *y*-intercept, if any exists.

62. Find the real zeros of f.

63. Solve $f(x) \geq 0$.

64. Find the number of solutions to $f(x) = 1$.

65. Does f appear to be even, odd, or neither?

66. List the intervals on which f is increasing.

67. List the intervals on which f is decreasing.

68. List the local maxima, if any exist.

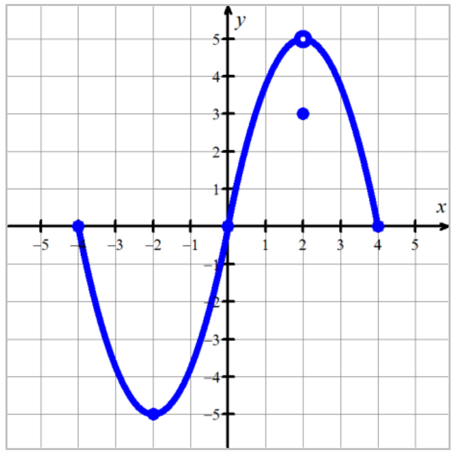
69. List the local minima, if any exist.

70. Find the absolute maximum, if it exists.

71. Find the absolute minimum, if it exists.

In Exercises 72 – 87, use the graph of $y = f(x)$ given below to answer the question.

Figure Ex1.2. 20



$y = f(x)$

72. Find the domain of f.

73. Find the range of f.

74. Determine $f(2)$.

75. Solve $f(x) = -5$.

76. List the *x*-intercepts, if any exist.

77. List the *y*-intercept, if any exists.

78. Find the real zeros of f.

79. Solve $f(x) \le 0$.

80. Find the number of solutions to $f(x) = 3$.

81. Does f appear to be even, odd, or neither?

82. List the intervals on which f is increasing.

83. List the intervals on which f is decreasing.

84. List the local maxima, if any exist.

85. List the local minima, if any exist.

86. Find the absolute maximum, if it exists.

87. Find the absolute minimum, if it exists.

88. The area A enclosed by a square, in square inches, is a function of the length of one of its sides x, when measured in inches. This relation is expressed by the formula $A(x) = x^2$ for $x > 0$. Find $A(3)$ and solve $A(x) = 36$. Interpret your answers to each. Why is x restricted to $x > 0$?

89. The area A enclosed by a circle, in square meters, is a function of its radius r, when measured in meters. This relation is expressed by the formula $A(r) = \pi r^2$ for $r > 0$. Find $A(2)$ and solve $A(r) = 16\pi$. Interpret your answers to each. Why is r restricted to $r > 0$?

90. The volume V enclosed by a cube, in cubic centimeters, is a function of the length of one of its sides x, when measured in centimeters. This relation is expressed by the formula $V(x)=x^3$ for $x>0$. Find $V(5)$ and solve $V(x)=27$. Interpret your answers to each. Why is x restricted to $x>0$?

91. The volume V enclosed by a sphere, in cubic feet, is a function of the radius of the sphere r, when measured in feet. This relation is expressed by the formula $V(r)=\frac{4\pi}{3}r^3$ for $r>0$. Find $V(3)$ and solve $V(r)=\frac{32\pi}{3}$. Interpret your answers to each. Why is r restricted to $r>0$?

92. The height of an object dropped from the roof of an eight story building is modeled by $h(t)=-16t^2+64$, $0\le t\le 2$. Here, h is the height of the object off the ground, in feet, t seconds after the object is dropped. Find $h(0)$ and solve $h(t)=0$. Interpret your answers to each. Why is t restricted to $0\le t\le 2$?

93. The temperature T, in degrees Fahrenheit, t hours after 6 AM is given by $T(t)=-\frac{1}{2}t^2+8t+3$ for $0\le t\le 12$. Find and interpret $T(0)$, $T(6)$, and $T(12)$.

94. The function $C(x)=x^2+10x+27$ models the cost, in hundreds of dollars, to produce x thousand pens. Find and interpret $C(0)$, $C(2)$, and $C(5)$.

95. Using data from the Bureau of Transportation Statistics, the average fuel economy F, in miles per gallon, for passenger cars in the US can be modeled by $F(t)=-0.0076t^2+0.45t+16$, $0\le t\le 28$, where t is the number of years since 1980. Use your calculator to find $F(0)$, $F(14)$, and $F(28)$. Round your answers to two decimal places and interpret your answers to each.

96. The population of Sasquatch in Portage County can be modeled by the function $P(t)=\frac{150t}{t+15}$, where t represents the number of years since 1803. Find and interpret $P(0)$ and $P(205)$. Discuss with your classmates what the applied domain and range of P should be.

97. For n copies of the book *Me and my Sasquatch*, a print on demand company charges $C(n)$ dollars, where $C(n)$ is determined by the formula

$$C(n)=\begin{cases} 15n & \text{if} \quad 1 \le n \le 25 \\ 13.50n & \text{if} \quad 25 < n \le 50 \\ 12n & \text{if} \quad n > 50 \end{cases}$$

(a) Find and interpret $C(20)$.

(b) How much does it cost to order 50 copies of the book? What about 51 copies?

(c) Your answer to part (b) should get you thinking. Suppose a bookstore estimates it will sell 50 copies of the book. How many books can, in fact, be ordered for the same price as those 50 copies? (Round your answer to a whole number of books.)

98. An on-line comic book retailer charges shipping costs according to the formula

$$S(n)=\begin{cases} 1.5n+2.5 & \text{if} \quad 1 \le n \le 14 \\ 0 & \text{if} \quad n \ge 15 \end{cases}$$

where n is the number of comic books purchased and $S(n)$ is the shipping cost in dollars.

(a) What is the cost to ship 10 comic books?

(b) What is the significance of the formula $S(n)=0$ for $n \ge 15$?

99. The cost C (in dollars) to talk m minutes a month on a mobile phone plan is modeled by

$$C(m)=\begin{cases} 25 & \text{if} \quad 0 \le m \le 1000 \\ 25+0.1(m-1000) & \text{if} \quad \text{m>1000} \end{cases}$$

(a) How much does it cost to talk 750 minutes per month with this plan?

(b) How much does it cost to talk 20 hours a month with this plan?

(c) Explain the terms of the plan in words.

100. We define the set of **integers** as $\mathbb{Z}=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$.[16] The **greatest integer of *x***, denoted by $\lfloor x \rfloor$, is defined to be the largest integer *k* with $k \le x$.

(a) Find $\lfloor 0.785 \rfloor$, $\lfloor 117 \rfloor$, $\lfloor -2.001 \rfloor$, and $\lfloor \pi+6 \rfloor$.

[16] The use of the letter $\mathbb{Z}$ for the integers is ostensibly because the German word *zahlen* means 'to count'.

(b) Discuss with your classmates how $\lfloor x \rfloor$ may be described as a piecewise defined function.

HINT: There are infinitely many pieces!

(c) Is $\lfloor a+b \rfloor = \lfloor a \rfloor + \lfloor b \rfloor$ always true? What if a or b is an integer? Test some values, make a conjecture, and explain your results.

101. Let $f(x) = \lfloor x \rfloor$ be the greatest integer function as defined in the last exercise.

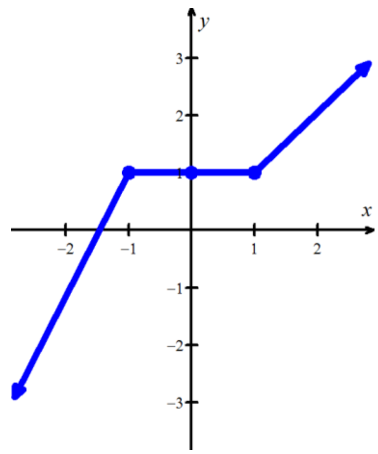
(a) Graph $y = f(x)$. Be careful to correctly describe the behavior of the graph near the integers.

(b) Is f even, odd or neither? Explain.

(c) Discuss with your classmates which points on the graph are local minimums, local maximums, or both. Is f ever increasing? Decreasing? Constant?

102. Consider the graph of the function f given below.

Figure Ex1.2. 21



$y = f(x)$

Refer back to **Definition 1.9** before answering the following.

(a) Show that f has a local maximum, but not a local minimum, at the point $(-1,1)$.

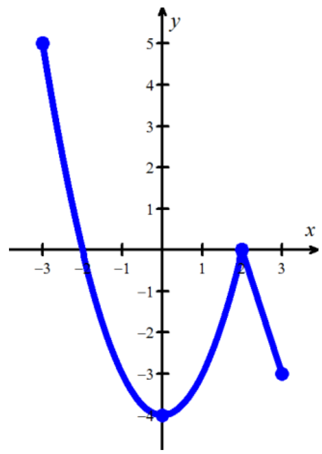
(b) Show that f has a local minimum, but not a local maximum, at the point $(1,1)$.

(c) Show that f has a local maximum AND a local minimum at the point $(0,1)$.

(d) Show that f is constant on the interval $(-1,1)$ and thus has both a local maximum AND a local minimum at every point $(x, f(x))$ where $-1 < x < 1$.

103. Using **Example 1.2.5** as a guide, show that the function g whose graph is given below does not have a local maximum at $(-3,5)$; nor does it have a local minimum at $(3,-3)$. Find its extrema, both local and absolute. What is unique about the point $(0,-4)$ on this graph? Also find the intervals on which g is increasing and the intervals on which g is decreasing.

Figure Ex1.2. 22



$y = g(x)$

1.3 Exercises

1. When examining the formula of a function that is the result of multiple transformations, how can you distinguish between a horizontal shift and a vertical shift?

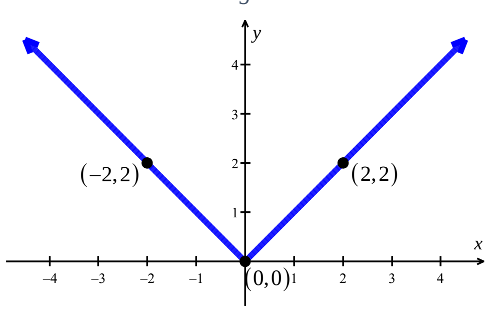
2. When examining the formula of a function that is the result of multiple transformations, how can you distinguish between a reflection across the *x*-axis and a reflection across the *y*-axis?

Suppose $(2,-3)$ is on the graph of $y=f(x)$. In Exercises 3 – 20, use the point $(2,-3)$ to find a point on the graph of the given transformed function.

3. $y=f(x)+3$
4. $y=f(x+3)$
5. $y=f(x)-1$
6. $y=f(x-1)$
7. $y=3f(x)$
8. $y=f(3x)$
9. $y=-f(x)$
10. $y=f(-x)$
11. $y=f(x-3)+1$
12. $y=2f(x+1)$
13. $y=10-f(x)$
14. $y=3f(2x)-1$
15. $y=\frac{1}{2}f(4-x)$
16. $y=5f(2x+1)+3$
17. $y=2f(1-x)-1$
18. $y=f\left(\frac{7-2x}{4}\right)$
19. $y=\frac{f(3x)-1}{2}$
20. $y=\frac{4-f(3x-1)}{7}$

The complete graph of $y=f(x)$ is given below. In Exercises 21 – 29, use it to sketch a graph of the given transformed function.

Figure Ex1.3. 1



The graph of $y=f(x)$ for Exercises 21 – 29

21. $y=f(x)+1$
22. $y=f(x)-2$
23. $y=f(x+1)$
24. $y=f(x-2)$
25. $y=2f(x)$
26. $y=f(2x)$

27. $y = 2 - f(x)$

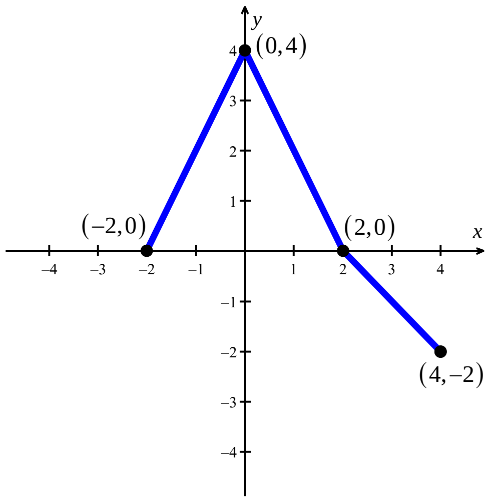
28. $y = f(2 - x)$

29. $y = 2 - f(2 - x)$

30. Some of the answers to Exercises 21 – 29 above should be the same. Which ones match up? What properties of the graph of $y = f(x)$ contribute to the duplication?

The complete graph of $y = f(x)$ is given below. In Exercises 31 – 39, use it to sketch a graph of the given transformed function.

Figure Ex1.3. 2



The graph of $y = f(x)$ for Exercises 31 – 39

31. $y = f(x) - 1$

32. $y = f(x+1)$

33. $y = \frac{1}{2} f(x)$

34. $y = f(2x)$

35. $y = -f(x)$

36. $y = f(-x)$

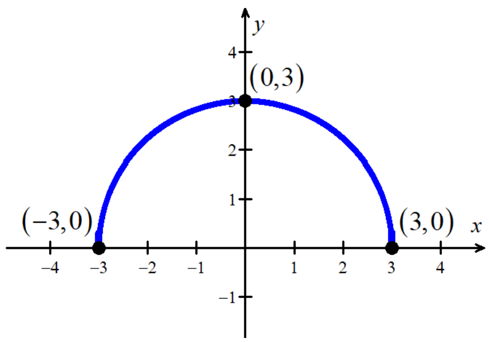
37. $y = f(x+1) - 1$

38. $y = 1 - f(x)$

39. $y = \frac{1}{2} f(x+1) - 1$

The complete graph of $y = f(x)$ is given below. In Exercises 40 – 51, use it to sketch a graph of the given transformed function.

Figure Ex1.3. 3



The graph of $y = f(x)$ for Exercises 40 – 51

40. $g(x) = f(x) + 3$

41. $h(x) = f(x) - \frac{1}{2}$

42. $j(x) = f\left(x - \frac{2}{3}\right)$

43. $a(x) = f(x+4)$

44. $b(x) = f(x+1) - 1$

45. $c(x) = \frac{3}{5} f(x)$

46. $d(x) = -2f(x)$

47. $k(x) = f\left(\frac{2}{3}x\right)$

48. $m(x) = -\frac{1}{4} f(3x)$

49. $n(x) = 4f(x-3) - 6$

50. $p(x) = 4 + f(1-2x)$

51. $q(x) = -\frac{1}{2} f\left(\frac{x+4}{2}\right) - 3$

52. Write a formula for a function g whose graph is obtained from $f(x) = \sqrt{x}$ after the sequence of transformations: (1) shift right 2 units; (2) shift down 3 units.

53. Write a formula for a function g whose graph is obtained from $f(x) = \sqrt{x}$ after the sequence of transformations: (1) shift down 3 units; (2) shift right 2 units.

54. Write a formula for a function g whose graph is obtained from $f(x) = \sqrt{x}$ after the sequence of transformations: (1) reflect across the x-axis; (2) shift up 1 unit.

55. Write a formula for a function g whose graph is obtained from $f(x) = \sqrt{x}$ after the sequence of transformations: (1) shift up 1 unit; (2) reflect across the x-axis.

56. Write a formula for a function g whose graph is obtained from $f(x) = \sqrt{x}$ after the sequence of transformations: (1) shift left 1 unit; (2) reflect across the y-axis; (3) shift up 2 units.

57. Write a formula for a function g whose graph is obtained from $f(x)=\sqrt{x}$ after the sequence of transformations: (1) reflect across the *y*-axis; (2) shift left 1 unit; (3) shift up 2 units.

58. Write a formula for a function g whose graph is obtained from $f(x)=\sqrt{x}$ after the sequence of transformations: (1) shift left 3 units; (2) scale vertically by a factor of 2; (3) shift down 4 units.

59. Write a formula for a function g whose graph is obtained from $f(x)=\sqrt{x}$ after the sequence of transformations: (1) shift left 3 units; (2) shift down 4 units; (3) scale vertically by a factor of 2.

60. Write a formula for a function g whose graph is obtained from $f(x)=\sqrt{x}$ after the sequence of transformations: (1) shift right 3 units; (2) scale horizontally by a factor of $\frac{1}{2}$; (3) shift up 1 unit.

61. Write a formula for a function g whose graph is obtained from $f(x)=\sqrt{x}$ after the sequence of transformations: (1) scale horizontally by a factor of $\frac{1}{2}$; (2) shift right 3 units; (3) shift up 1 unit.

62. Write a formula for a function g whose graph is obtained from $f(x)=|x|$ after the sequence of transformations: (1) shift down 3 units; (2) shift right 1 unit.

63. Write a formula for a function g whose graph is obtained from $f(x)=\frac{1}{x}$ after the sequence of transformations: (1) shift down 4 units; (2) shift right 3 units.

64. Write a formula for a function g whose graph is obtained from $f(x)=\frac{1}{x^2}$ after the sequence of transformations: (1) shift up 2 units; (2) shift left 4 units.

65. Write a formula for a function g whose graph is obtained from $f(x)=|x|$ after the sequence of transformations: (1) reflect across the *y*-axis; (2) scale horizontally by a factor of $\frac{1}{4}$.

66. Write a formula for a function g whose graph is obtained from $f(x)=\frac{1}{x^2}$ after the sequence of transformations: (1) scale vertically by a factor of $\frac{1}{3}$; (2) shift left 2 units; (3) shift down 3 units.

67. Write a formula for a function g whose graph is obtained from $f(x)=\frac{1}{x}$ after the sequence of transformations: (1) scale vertically by a factor of 8; (2) shift right 4 units; (3) shift up 2 units.

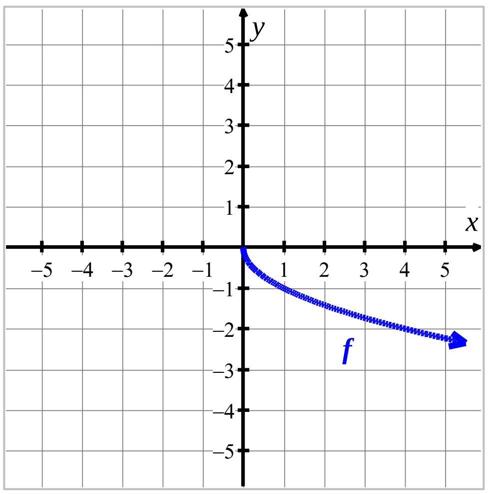
68. Write a formula for a function g whose graph is obtained from $f(x)=x^2$ after the sequence of transformations: (1) scale vertically by a factor of $\frac{1}{2}$; (2) shift right 5 units; (3) shift up 1 unit.

69. Write a formula for a function g whose graph is obtained from $f(x)=x^2$ after the sequence of transformations: (1) scale horizontally by a factor of $\frac{1}{3}$; (2) shift left 4 units; (3) shift down 3 units.

In Exercises 70 – 79, use the graphs of the transformed toolkit functions to write a formula for each of the resulting functions.

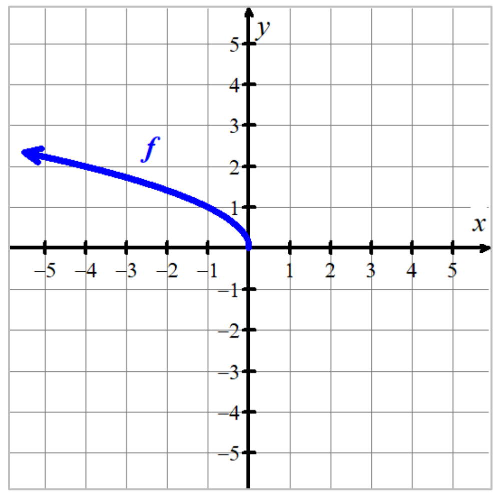
70.

Figure Ex1.3. 4



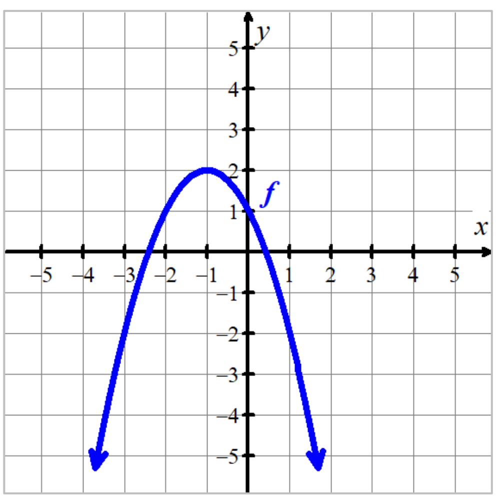
71.

Figure Ex1.3. 5



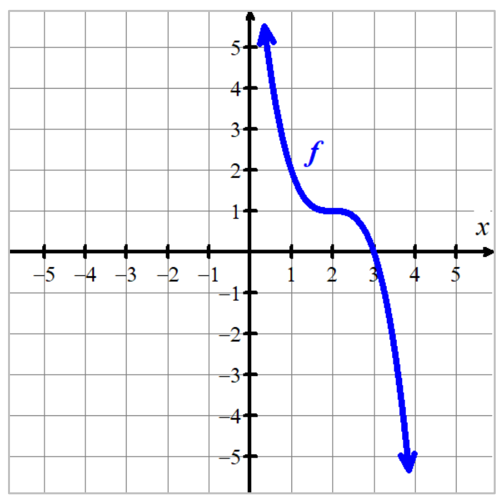
72.

Figure Ex1.3. 6



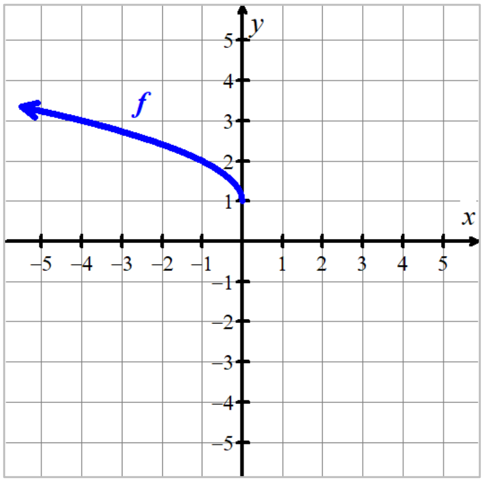
73.

Figure Ex1.3. 7



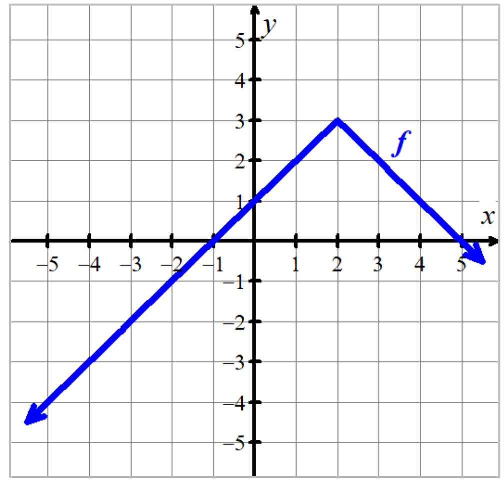
74.

Figure Ex1.3. 8



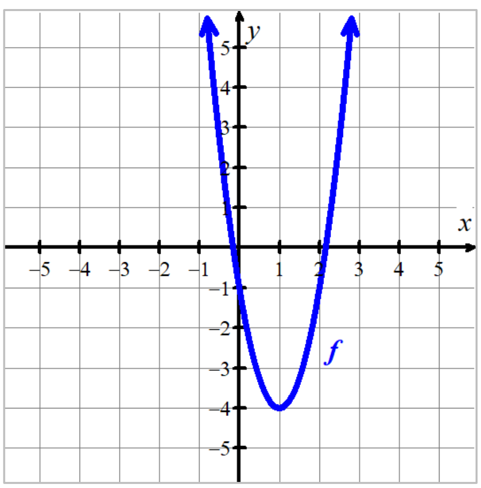
75.

Figure Ex1.3. 9



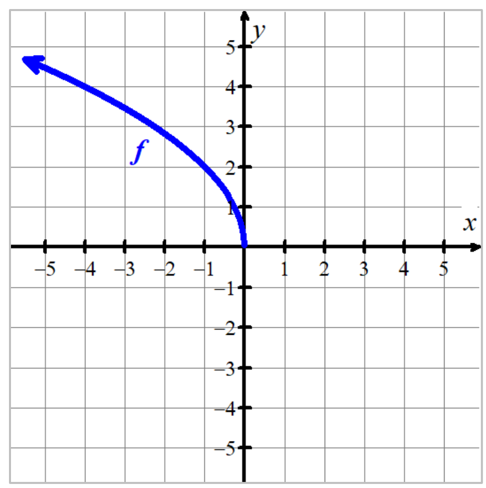
76.

Figure Ex1.3. 10



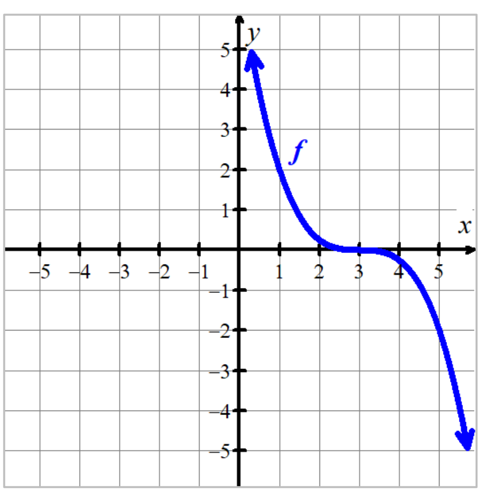
77.

Figure Ex1.3. 11



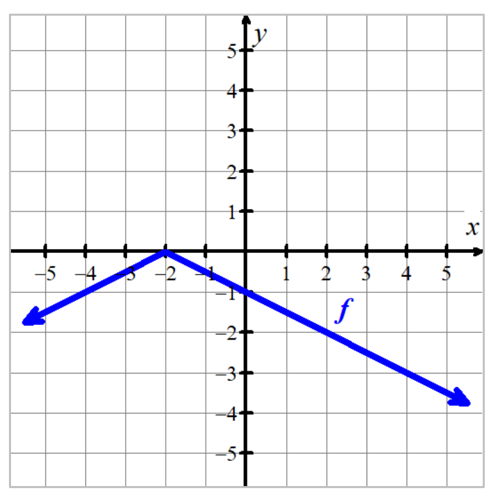
78.

Figure Ex1.3. 12



79.

Figure Ex1.3. 13



In Exercises 80 – 92, sketch a graph of the function as a transformation of the graph of one of the toolkit functions.

80. $f(x)=(x+1)^2-3$

81. $h(x)=|x-1|+4$

82. $k(x)=(x-2)^3-1$

83. $m(x)=3+\sqrt{x+2}$

84. $g(x)=4(x+1)^2-5$

85. $g(x)=5(x+3)^2-2$

86. $h(x)=-2|x-4|+3$

87. $k(x)=-3\sqrt{x}-1$

88. $m(x)=\frac{1}{2}x^3$

89. $n(x)=\frac{1}{3}|x-2|$

90. $p(x)=\left(\frac{1}{3}x\right)^3-3$

91. $q(x)=\left(\frac{1}{4}x\right)^3+1$

92. $a(x)=\sqrt{-x+4}$

93. For many common functions, the properties of algebra make a horizontal scaling the same as a vertical scaling by (possibly) a different factor. For example, we stated earlier that $\sqrt{9x}=3\sqrt{x}$. With the help of your classmates, find the equivalent vertical scaling produced by the horizontal scalings $y=(2x)^3$, $y=|5x|$, $y=\sqrt[3]{27x}$ and $y=\left(\frac{1}{2}x\right)^2$. What about $y=(-2x)^3$, $y=|-5x|$, $y=\sqrt[3]{-27x}$ and $y=\left(-\frac{1}{2}x\right)^2$?

94. As mentioned earlier in the section, in general, the order in which transformations are applied matters. Yet, in one of our examples with two transformations, the order did not matter. With the help of your classmates, determine the situations in which order does matter and those in which it does not.

95. What happens if you reflect an even function across the *y*-axis?

96. What happens if you reflect an odd function across the *y*-axis?

97. What happens if you reflect an even function across the *x*-axis?

98. What happens if you reflect an odd function across the *x*-axis?

99. How would you describe symmetry about the origin in terms of reflections?

1.5 Exercises

1. Why do we restrict the domain of the function $f(x)=x^2$ to find the function's inverse?

2. Are one-to-one functions either always increasing or always decreasing? Why or why not?

In Exercises 3 – 4, use function composition to verify that $f(x)$ and $g(x)$ are inverse functions.

3. $f(x)=\sqrt[3]{x-1}$ and $g(x)=x^3+1$

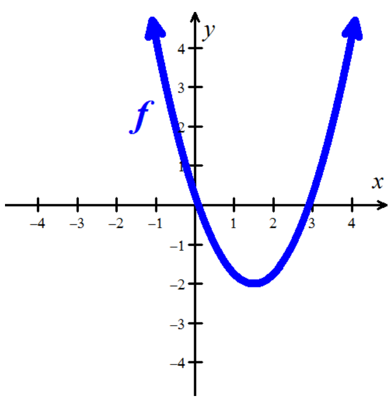
4. $f(x)=-3x+5$ and $g(x)=\dfrac{x-5}{-3}$

5. Show that the function $f(x)=3-x$ is its own inverse.

In Exercises 6 – 7, determine whether the graph represents a one-to-one function.

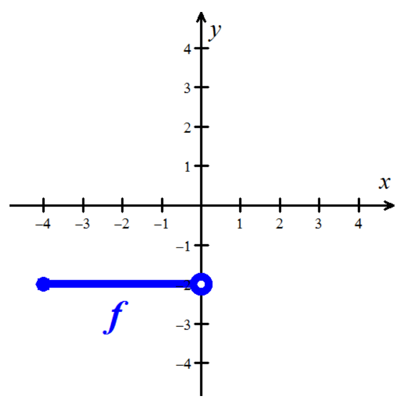
6.

Figure Ex1.5. 1



7.

Figure Ex1.5. 2



8. Determine if the table of values represents y as a function of x. If the table does represent y as a function of x, is the function one-to-one?

(a)

x	4	10	15
y	2	7	7

(b)

x	4	10	15
y	2	7	13

(c)

x	4	10	10
y	2	7	13

9. Use the following table of values for the function $f(x)$ to create a table of values for the function's inverse, $f^{-1}(x)$.

x	1	5	8	13	16
$f(x)$	2	4	9	12	15

10. Use the following table of values for the function $f(x)$ to determine the requested values.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	8	0	9	7	2	3	4	1	6	5

a) What is $f(7)$?

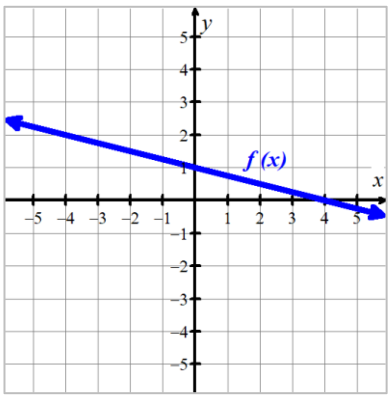
b) If $f(x)=3$, then what is x?

c) What is $f^{-1}(1)$?

d) If $f^{-1}(x)=5$, then what is x?

11. Use the following graph of the function $f(x)$ to determine the requested values.

Figure Ex1.5. 3



a) What is $f(0)$?

b) If $f(x)=0$, then what is x?

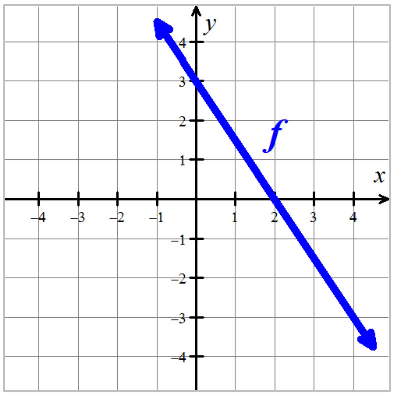
c) What is $f^{-1}(0)$?

d) If $f^{-1}(x)=0$, then what is x?

In Exercises 12 – 15, sketch the graph of the inverse function.

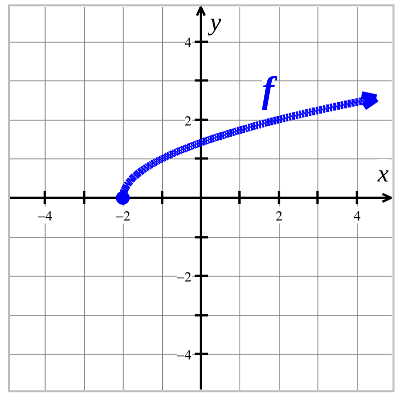
12.

Figure Ex1.5. 4



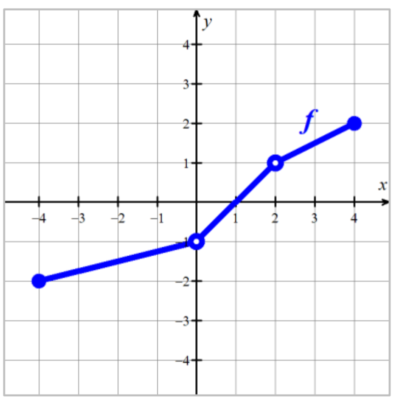
13.

Figure Ex1.5. 5



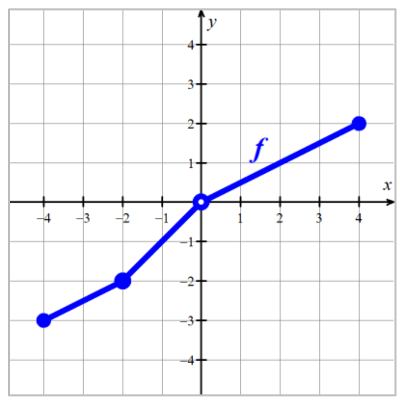
14.

Figure Ex1.5. 6



15.

Figure Ex1.5. 7



In Exercises 16 – 25, show that the given function is one-to-one and find its inverse. Check your answer algebraically and graphically. Verify that the range of f is the domain of f^{-1} and vice-versa.

16. $f(x) = 6x - 2$

17. $f(x) = 42 - x$

18. $f(x) = \dfrac{x-2}{3} + 4$

19. $f(x) = 1 - \dfrac{4+3x}{5}$

20. $f(x) = \sqrt{3x-1} + 5$

21. $f(x) = 2 - \sqrt{x-5}$

22. $f(x) = 3\sqrt{x-1} - 4$

23. $f(x) = 1 - 2\sqrt{2x+5}$

24. $f(x) = 3(x+4)^2 - 5, \;\; x \le -4$

25. $f(x) = \dfrac{3}{4-x}$

In Exercises 26 – 35, find the inverse of the given one-to-one function.

26. $f(x)=\sqrt[5]{3x-1}$

27. $f(x)=3-\sqrt[3]{x-2}$

28. $f(x)=x^2-10x,\ x\geq 5$

29. $f(x)=x^2-6x+5,\ x\leq 3$

30. $f(x)=4x^2+4x+1,\ x<-1$

31. $f(x)=\dfrac{x}{1-3x}$

32. $f(x)=\dfrac{2x-1}{3x+4}$

33. $f(x)=\dfrac{4x+2}{3x-6}$

34. $f(x)=\dfrac{-3x-2}{x+3}$

35. $f(x)=\dfrac{x-2}{2x-1}$

36. If $f(x)=\dfrac{x+5}{x+6}$, then what is $f^{-1}(-5)$? How can you determine the value of $f^{-1}(-5)$ without finding $f^{-1}(x)$?

With the help of your classmates, find the inverses of the functions in Exercises 37 – 40.

37. $f(x)=a\,x+b,\ a\neq 0$

38. $f(x)=a\sqrt{x-h}+k,\ a\neq 0,\ x\geq h$

39. $f(x)=a\,x^2+b\,x+c$ where $a\neq 0,\ x\geq -\dfrac{b}{2a}$

40. $f(x)=\dfrac{a\,x+b}{c\,x+d}$ (See **Exercise 46** below.)

41. Show that the Fahrenheit to Celsius conversion function, $C(F)=\frac{5}{9}(F-32)$, is invertible and that its inverse is $F(C)=\frac{9}{5}C+32$, the Celsius to Fahrenheit conversion formula.

42. A car travels at a constant speed of 50 miles per hour. The distance the car travels in miles is a function of time, t, in hours given by $d(t)=50t$. Find the inverse function by expressing the time of travel in terms of the distance traveled. Call this function $t(d)$. Find $t(180)$ and interpret its meaning.

43. The circumference C of a circle is a function of its radius r given by $C(r)=2\pi r$. Express the radius of a circle as a function of its circumference. Call this function $r(C)$. Find $r(36\pi)$ and interpret its meaning.

44. With the help of your classmates, explain why a function which is either strictly increasing or strictly decreasing on its entire domain would have to be one-to-one, hence invertible.

45. What graphical feature must a function f possess for it to be its own inverse?

46. What conditions must you place on the values of a, b, c, and d in **Exercise 40** in order to guarantee that the function is invertible?

1.4 Exercises

1. If the order is reversed when composing two functions, can the result ever be the same as the answer in the original order of the composition? If yes, give an example. If no, explain why not.

2. How do you find the domain for the composition of two functions, $f \circ g$?

In Exercises 3 – 8, use the given function f to find and simplify the following:

(a) $f(4x)$ (b) $4f(x)$ (c) $f(-x)$

(d) $f(x-4)$ (e) $f(x)-4$ (f) $f(x^2)$

3. $f(x)=2x+1$
4. $f(x)=3-4x$
5. $f(x)=2-x^2$
6. $f(x)=x^2-3x+2$
7. $f(x)=\dfrac{x}{x-1}$
8. $f(x)=\dfrac{2}{x^3}$

In Exercises 9 – 16, use the given function f to find and simplify the following:

(a) $f(2a)$ (b) $2f(a)$ (c) $f\left(\dfrac{2}{a}\right)$

(d) $\dfrac{f(a)}{2}$ (e) $f(a+h)$ (f) $f(a)+f(h)$

9. $f(x)=2x-5$
10. $f(x)=5-2x$
11. $f(x)=2x^2-1$
12. $f(x)=3x^2+3x-2$
13. $f(x)=\sqrt{2x+1}$
14. $f(x)=117$
15. $f(x)=\dfrac{x}{2}$
16. $f(x)=\dfrac{2}{x}$

In Exercises 17 – 26, use the pair of functions f and g to find the following values, if they exist.

(a) $(f+g)(2)$ (b) $(f-g)(-1)$ (c) $(g-f)(1)$

(d) $(f\cdot g)\left(\dfrac{1}{2}\right)$ (e) $\left(\dfrac{f}{g}\right)(0)$ (f) $\left(\dfrac{g}{f}\right)(-2)$

17. $f(x)=3x+1,\ g(x)=4-x$
18. $f(x)=x^2,\ g(x)=-2x+1$

19. $f(x)=x^2-x,\ g(x)=12-x^2$

20. $f(x)=2x^3,\ g(x)=-x^2-2x-3$

21. $f(x)=\sqrt{x+3},\ g(x)=2x-1$

22. $f(x)=\sqrt{4-x},\ g(x)=\sqrt{x+2}$

23. $f(x)=2x,\ g(x)=\dfrac{1}{2x+1}$

24. $f(x)=x^2,\ g(x)=\dfrac{3}{2x-3}$

25. $f(x)=x^2,\ g(x)=\dfrac{1}{x^2}$

26. $f(x)=x^2+1,\ g(x)=\dfrac{1}{x^2+1}$

In Exercises 27 – 36, use the pair of functions f and g to find and simplify an expression for the indicated function. Determine the domain.

(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(f\cdot g)(x)$ (d) $\left(\dfrac{f}{g}\right)(x)$

27. $f(x)=2x+1,\ g(x)=x-2$

28. $f(x)=1-4x,\ g(x)=2x-1$

29. $f(x)=x^2,\ g(x)=3x-1$

30. $f(x)=x^2-x,\ g(x)=7x$

31. $f(x)=x^2-4,\ g(x)=3x+6$

32. $f(x)=-x^2+x+6,\ g(x)=x^2-9$

33. $f(x)=\dfrac{x}{2},\ g(x)=\dfrac{2}{x}$

34. $f(x)=x-1,\ g(x)=\dfrac{1}{x-1}$

35. $f(x)=x,\ g(x)=\sqrt{x+1}$

36. $f(x)=\sqrt{x-5},\ g(x)=f(x)=\sqrt{x-5}$

In Exercises 37 – 54, find and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the given function.

37. $f(x)=2x-5$

38. $f(x)=-3x+5$

39. $f(x)=6$

40. $f(x)=3x^2-x$

41. $f(x)=-x^2+2x-1$

42. $f(x)=4x^2$

43. $f(x)=x-x^2$

44. $f(x)=x^3+1$

45. $f(x)=mx+b$ where $m\neq 0$

46. $f(x)=ax^2+bx+c$ where $a\neq 0$

47. $f(x)=\dfrac{2}{x}$

48. $f(x)=\dfrac{3}{1-x}$

49. $f(x) = \frac{1}{x^2}$

50. $f(x) = \frac{2}{x+5}$

51. $f(x) = \frac{1}{4x-3}$

52. $f(x) = \frac{3x}{x+1}$

53. $f(x) = \frac{x}{x-9}$

54. $f(x) = \frac{x^2}{2x+1}$

In Exercises 55 – 69, use the following table of function values to compute the indicated value, if it exists.

x	-3	-2	-1	0	1	2	3
$f(x)$	4	2	0	1	3	4	-1
$g(x)$	-2	0	-4	0	-3	1	2

55. $(f+g)(-3)$

56. $(f \cdot g)(-1)$

57. $(g-f)(3)$

58. $\left(\frac{f}{g}\right)(-1)$

59. $\left(\frac{f}{g}\right)(2)$

60. $\left(\frac{g}{f}\right)(-1)$

61. $(f \circ g)(3)$

62. $f(g(-1))$

63. $(f \circ f)(0)$

64. $g(f(-3))$

65. $(g \circ g)(-2)$

66. $(g \circ f)(-2)$

67. $f(f(f(-1)))$

68. $f(f(f(f(f(1)))))$

69. $(\underbrace{g \circ g \circ \cdots \circ g}_{\text{n times}})(0)$

In Exercises 70 – 81, use the given pair of functions to find the following values, if they exist.

(a) $(g \circ f)(0)$

(b) $(f \circ g)(-1)$

(c) $(f \circ f)(2)$

(d) $(g \circ f)(-3)$

(e) $(f \circ g)\left(\frac{1}{2}\right)$

(f) $(f \circ f)(-2)$

70. $f(x) = x^2$, $g(x) = 2x+1$

71. $f(x) = 4-x$, $g(x) = 1-x^2$

72. $f(x) = 4-3x$, $g(x) = |x|$

73. $f(x) = |x-1|$, $g(x) = x^2-5$

74. $f(x) = 4x+5$, $g(x) = \sqrt{x}$

75. $f(x) = \sqrt{3-x}$, $g(x) = x^2+1$

76. $f(x) = 6-x-x^2$, $g(x) = x\sqrt{x+10}$

77. $f(x) = \sqrt[3]{x+1}$, $g(x) = 4x^2-x$

78. $f(x) = \frac{3}{1-x}$, $g(x) = \frac{4x}{x^2+1}$

79. $f(x) = \frac{x}{x+5}$, $g(x) = \frac{2}{7-x^2}$

80. $f(x)=\dfrac{2x}{5-x^2}$, $g(x)=\sqrt{4x+1}$

81. $f(x)=\sqrt{2x+5}$, $g(x)=\dfrac{10x}{x^2+1}$

In Exercises 82 – 91, use the given pair of functions to find and simplify expressions for the following functions. State the domain of each using interval notation.

(a) $(g\circ f)(x)$ (b) $(f\circ g)(x)$ (c) $(f\circ f)(x)$

82. $f(x)=2x+3$, $g(x)=x^2-9$

83. $f(x)=x^2-x+1$, $g(x)=3x-5$

84. $f(x)=x^2-4$, $g(x)=|x|$

85. $f(x)=3x-5$, $g(x)=\sqrt{x}$

86. $f(x)=|x+1|$, $g(x)=\sqrt{x}$

87. $f(x)=|x|$, $g(x)=\sqrt{4-x}$

88. $f(x)=\dfrac{1}{x}$, $g(x)=x-3$

89. $f(x)=3x-1$, $g(x)=\dfrac{1}{x+3}$

90. $f(x)=\dfrac{2}{x}$, $g(x)=\dfrac{3}{x+5}$

91. $f(x)=\dfrac{x}{2x+1}$, $g(x)=\dfrac{2x+1}{x}$

92. Use the functions $f(x)=\dfrac{1}{x^2-8}$ and $g(x)=\sqrt{x+2}$ to find and simplify $(f\circ g)(x)$.

In Exercises 93 – 96, use the given pair of functions to find and simplify expressions for the following composite functions. You are not required to find the domain.

(a) $(g\circ f)(x)$ (b) $(f\circ g)(x)$ (c) $(f\circ f)(x)$

93. $f(x)=3-x^2$, $g(x)=\sqrt{x+1}$

94. $f(x)=x^2-x-1$, $g(x)=\sqrt{x-5}$

95. $f(x)=\dfrac{2x}{x^2-4}$, $g(x)=\sqrt{1-x}$

96. $f(x)=\sqrt{2-4x}$, $g(x)=-\dfrac{3}{x}$

In Exercises 97 – 102, use $f(x)=-2x$, $g(x)=\sqrt{x}$, and $h(x)=|x|$ to find and simplify expressions for the following composite functions. State the domain of each using interval notation.

97. $(h\circ g\circ f)(x)$ 98. $(h\circ f\circ g)(x)$ 99. $(g\circ f\circ h)(x)$

100. $(g\circ h\circ f)(x)$ 101. $(f\circ h\circ g)(x)$ 102. $(f\circ g\circ h)(x)$

In Exercises 103 – 114, write the given function as a composition of two or more non-identity[23] functions. (There are several correct answers so check your answer using function composition.)

103. $p(x) = (2x+3)^3$

104. $P(x) = (x^2 - x + 1)^5$

105. $h(x) = \sqrt{2x-1}$

106. $H(x) = |7-3x|$

107. $r(x) = \dfrac{2}{5x+1}$

108. $R(x) = \dfrac{7}{x^2-1}$

109. $q(x) = \dfrac{|x|+1}{|x|-1}$

110. $Q(x) = \dfrac{2x^3+1}{x^3-1}$

111. $v(x) = \dfrac{2x+1}{3-4x}$

112. $V(x) = \dfrac{x^2}{x^4+1}$

113. $w(x) = \dfrac{1}{(x-2)^3}$

114. $W(x) = \left(\dfrac{1}{2x-3}\right)^2$

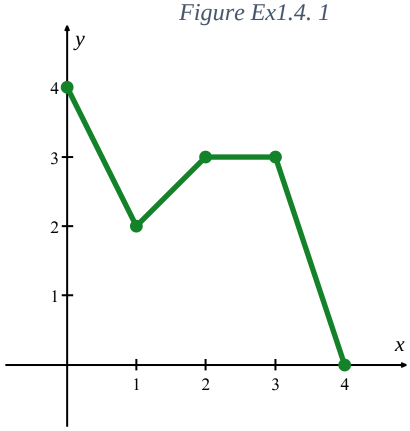
115. Write the function $F(x) = \sqrt{\dfrac{x^3+6}{x^3-9}}$ as a composition of three or more non-identity functions.

116. Let $g(x) = -x$, $h(x) = x+2$, $j(x) = 3x$, and $k(x) = x-4$. In what order must these functions be composed with $f(x) = \sqrt{x}$ to create $F(x) = 3\sqrt{-x+2} - 4$?

117. What linear functions could be used to transform $f(x) = x^3$ into $F(x) = -\frac{1}{2}(2x-7)^3 + 1$? What is the proper order of composition?

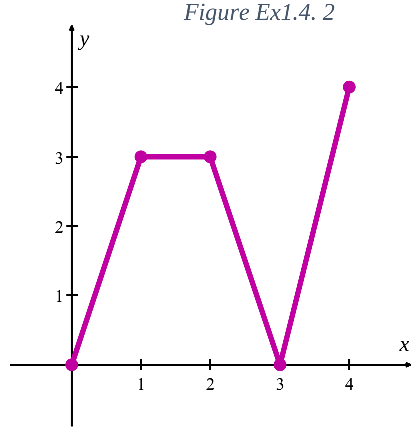
In Exercises 118 – 123, use the graphs of $y = f(x)$ and $y = g(x)$ below to find the function value.

Figure Ex1.4. 1



$y = f(x)$

Figure Ex1.4. 2



$y = g(x)$

118. $(g \circ f)(1)$

119. $(f \circ g)(3)$

120. $(g \circ f)(2)$

[23] The identity function is $I(x) = x$.

121. $(f \circ g)(0)$ 122. $(f \circ f)(1)$ 123. $(g \circ g)(1)$

124. The volume V of a cube is a function of its side length x. Let's assume that $x = t + 1$ is also a function of time t, where x is measured in inches and t is measured in minutes. Find a formula for V as a function of t.

125. A store offers a 30% discount on the price x of selected items. Then, the store takes off an additional 15% at the cash register. Use function composition to find a price function $P(x)$ that computes the final price of the item in terms of the original price x.

126. A rain drop hitting a lake makes a circular ripple. If the radius, in inches, grows as a function of time in minutes according to $r(t) = 25\sqrt{t+2}$, find the area of the ripple as a function of time. Find the area of the ripple at $t = 2$.

127. Use the function you found in the previous exercise to find the area of the ripple after 5 minutes.

128. The number of bacteria in a refrigerated food product is given by $N(T) = 23T^2 - 56T + 1$, $3 < T < 33$, where T is the temperature of the food. When the food is removed from the refrigerator, the temperature is given by $T(t) = 5t + 1.5$, where t is the time in hours. Find the composite function $N(T(t))$, and use it to find the time when the bacteria count reaches 6752.

129. Discuss with your classmates how real-world processes such as filling out federal income tax forms or computing your final course grade could be viewed as a use of function composition. Find a process for which composition with itself (iteration) makes sense.