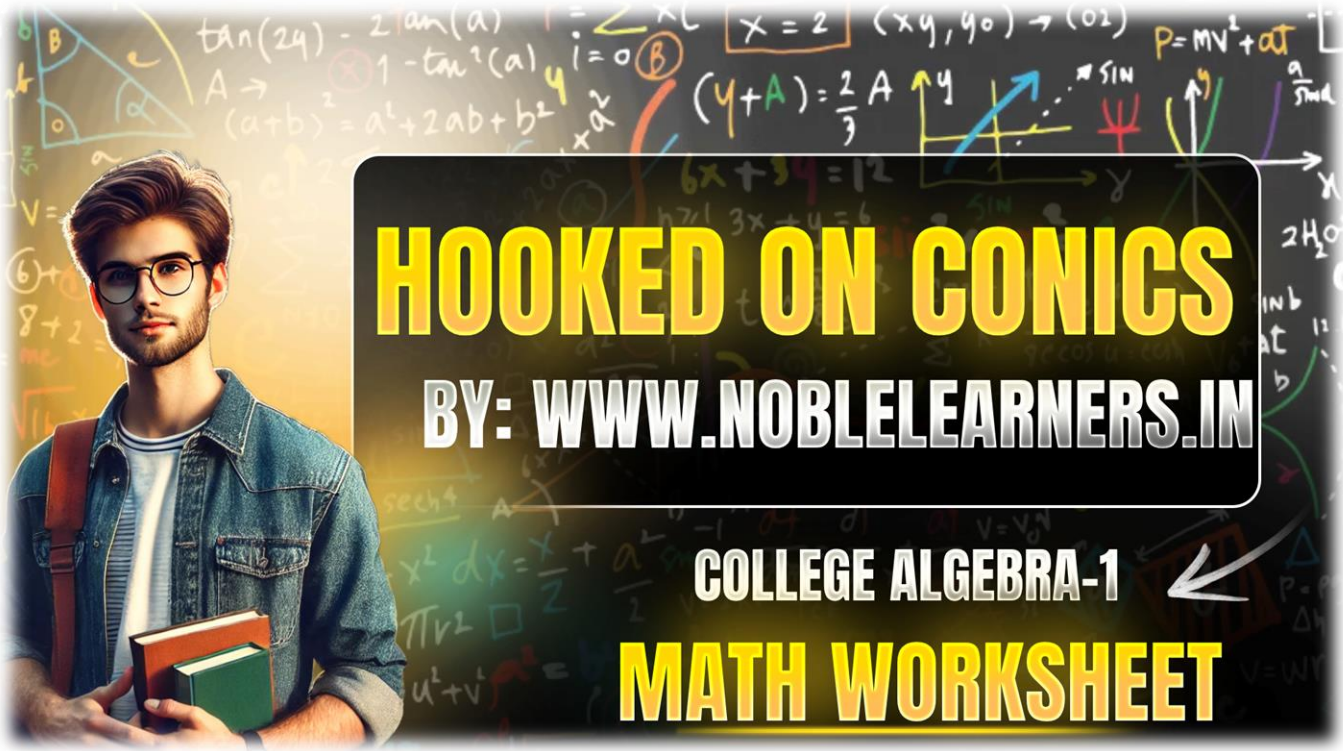

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5.2 Exercises

1. Define a circle in terms of its center.
2. In the standard equation of a circle, explain why r must be greater than 0.

In Exercises 3 – 8, write the equation in standard form if it represents a circle. If the equation does not represent a circle, explain how the equation violates the definition of a circle.

3. $x^2-4x+y^2+10y=-25$
4. $-2x^2-36x-2y^2-112=0$
5. $x^2+y^2+8x-10y-1=0$
6. $x^2+y^2+5x-y-1=0$
7. $4x^2+4y^2-24y+36=0$
8. $x^2+x+y^2-\frac{6}{5}y=1$

In Exercises 9 – 26, find the center and radius of the circle. Graph the circle.

9. $(x+5)^2+(y+3)^2=1$
10. $(x-2)^2+(y-3)^2=9$
11. $(x-4)^2+(y+2)^2=16$
12. $(x+2)^2+(y-5)^2=4$
13. $x^2+(y+2)^2=25$
14. $(x-1)^2+y^2=36$
15. $(x-1)^2+(y-3)^2=\frac{9}{4}$
16. $x^2+y^2=64$
17. $x^2+y^2=49$
18. $2x^2+2y^2=8$
19. $x^2+y^2+2x+6y+9=0$
20. $x^2+y^2-6x-8y=0$
21. $x^2+y^2-4x+10y-7=0$
22. $x^2+y^2+12x-14y+21=0$
23. $x^2+y^2+6y+5=0$
24. $x^2+y^2-10y=0$
25. $x^2+y^2+4x=0$
26. $x^2+y^2-14x+13=0$

In Exercises 27 – 40, put the equation of the circle that has the given properties into standard form.

27. Center $(-1,-5)$, Radius 10
28. Center $(4,-2)$, Radius 3
29. Center $\left(-3,\frac{7}{13}\right)$, Radius $\frac{1}{2}$
30. Center $(5,-9)$, Radius $\ln(8)$
31. Center $(-e,\sqrt{2})$, Radius π
32. Center (π,e^2), Radius $\sqrt[3]{91}$
33. Center $(3,5)$, containing the point $(-1,-2)$
34. Center $(3,6)$, containing the point $(-1,4)$
35. Center $(3,-2)$, containing the point $(3,6)$
36. Center $(6,-6)$, containing the point $(2,-3)$
37. Center $(4,4)$, containing the point $(2,2)$
38. Center $(-5,6)$, containing the point $(-2,3)$

39. Endpoints of a diameter are $(3,6)$ and $(-1,4)$

40. Endpoints of a diameter are $\left(\frac{1}{2},4\right)$ and $\left(\frac{3}{2},-1\right)$

41. The Giant Wheel at Cedar Point is a circle with diameter 128 feet which sits on an 8 foot tall platform, resulting in an overall height of 136 feet. Find an equation for the wheel assuming that its center lies on the *y*-axis and that the ground is the *x*-axis.

42. Verify that the following points lie on the Unit Circle: $(\pm1,0)$, $(0,\pm1)$, $\left(\pm\frac{\sqrt{2}}{2},\pm\frac{\sqrt{2}}{2}\right)$, $\left(\pm\frac{1}{2},\pm\frac{\sqrt{3}}{2}\right)$, and $\left(\pm\frac{\sqrt{3}}{2},\pm\frac{1}{2}\right)$.

43. The points $(-2,-2)$, $(1,1)$, $(4,-2)$, $(1,-5)$, $\left(0,-2\sqrt{2}-2\right)$, and $\left(0,2\sqrt{2}-2\right)$ lie on the circle $(x-1)^2+(y+2)^2=9$. Find three other points that lie on the circle.

5.3 Exercises

1. Define a parabola in terms of its focus and directrix.
2. If the equation of a parabola is written in standard form and p is positive and the directrix is a vertical line, then what can we conclude about the graph of the parabola?
3. If the equation of a parabola is written in standard form and p is negative and the directrix is a horizontal line, then what can we conclude about the graph of the parabola?
4. As the graph of a parabola becomes wider, what happens to the distance between the focus and the directrix?

In Exercises 5 – 8, write the equation in standard form if it represents a parabola. If the equation does not represent a parabola, explain how the equation violates the definition of a parabola.

5. $y^2 = 4 - x^2$
6. $y = 4x^2$
7. $y^2 + 12x - 6y - 51 = 0$
8. $3x^2 - 6y^2 = 12$

In Exercises 9 – 16, find the vertex, the focus, and the directrix of the parabola. Graph the parabola. Include the endpoints of the latus rectum in your sketch.

9. $(x-3)^2 = -16y$
10. $\left(x+\frac{7}{3}\right)^2 = 2\left(y+\frac{5}{2}\right)$
11. $(y-2)^2 = -12(x+3)$
12. $(y+4)^2 = 4x$
13. $(x-1)^2 = 4(y+3)$
14. $(x+2)^2 = -20(y-5)$
15. $(y-4)^2 = 18(x-2)$
16. $\left(y+\frac{3}{2}\right)^2 = -7\left(x+\frac{9}{2}\right)$

In Exercises 17 – 22, put the equation of the parabola in standard form. Find the vertex, the focus, and the directrix. Graph the parabola.

17. $y^2 - 10y - 27x + 133 = 0$
18. $25x^2 + 20x + 5y - 1 = 0$
19. $x^2 + 2x - 8y + 49 = 0$
20. $2y^2 + 4y + x - 8 = 0$
21. $x^2 - 10x + 12y + 1 = 0$
22. $3y^2 - 27y + 4x + \frac{211}{4} = 0$

In Exercises 23 – 31, find the standard form of the equation of the parabola that has the given properties.

23. Directrix $y = 4$, Focus $(0,-4)$

24. Directrix $x = 4$, Focus $(-4,0)$

25. Vertex $(-2,3)$, Directrix $x = -\frac{7}{2}$

26. Vertex $(1,2)$, Focus $\left(1,\frac{1}{3}\right)$

27. Vertex $(0,0)$ with endpoints of the latus rectum $(2,1)$ and $(-2,1)$

28. Vertex $(0,0)$ with endpoints of the latus rectum $(-2,4)$ and $(-2,-4)$

29. Vertex $(1,2)$ with endpoints of the latus rectum $(-5,5)$ and $(7,5)$

30. Vertex $(-3,-1)$ with endpoints of the latus rectum $(0,5)$ and $(0,-7)$

31. Vertex $(-8,-9)$, containing the points $(0,0)$ and $(-16,0)$

In Exercises 32 – 36, given the graph of the parabola, determine its equation.

32.

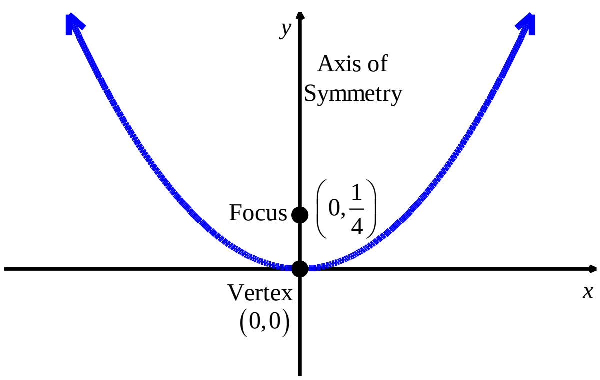

Figure Ex5.3. 1

33.

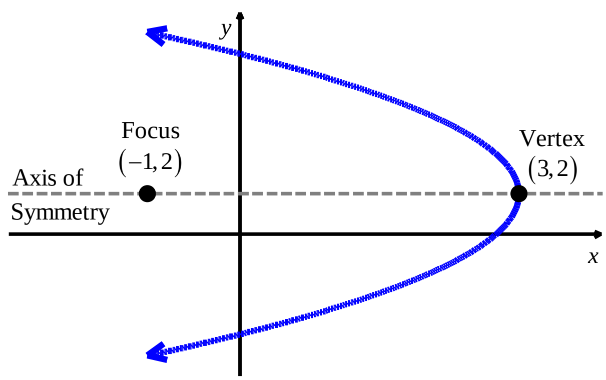

Figure Ex5.3. 2

34.

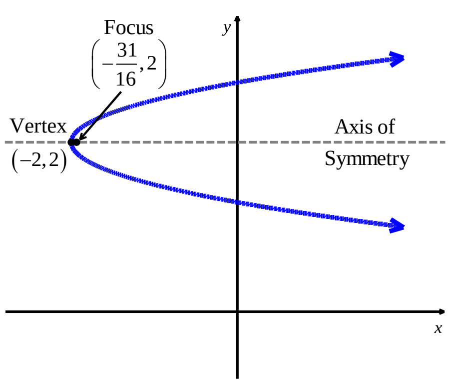

Figure Ex5.3. 3

35.

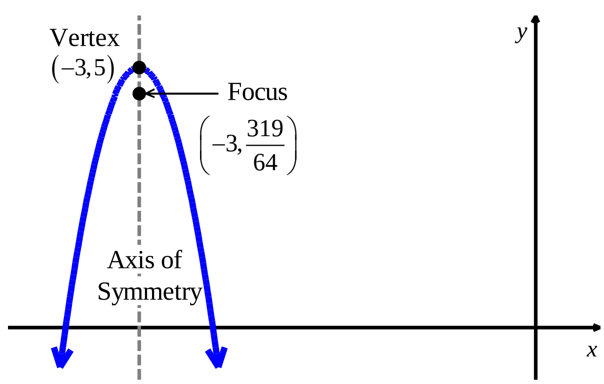

Figure Ex5.3. 4

36.

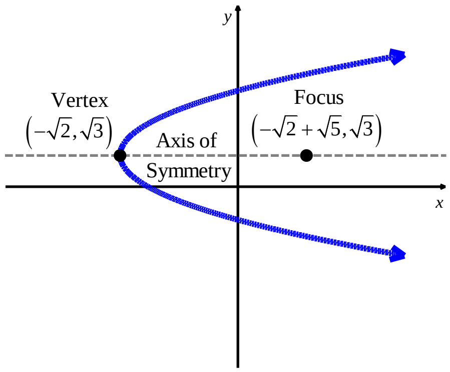

Figure Ex5.3. 5

37. The mirror in Carl's flashlight is a paraboloid. If the mirror is 5 centimeters in diameter and 2.5 centimeters deep, where should the light bulb be placed so it is at the focus of the mirror?

38. A parabolic Wi-Fi antenna is constructed by taking a flat sheet of metal and bending it into a parabolic shape. If the cross section of the antenna is a parabola which is 45 centimeters wide and 25 centimeters deep, where should the receiver be placed to maximize reception?

39. A parabolic arch is 6 feet wide at the base and 9 feet tall in the middle. Find the height of the arch exactly 1 foot in from the base of the arch.

40. A satellite dish is shaped like a paraboloid. The receiver is to be located at the focus. If the dish is 12 feet across at its opening and 4 feet deep at its center, where should the receiver be placed?

41. A searchlight is shaped like a paraboloid. A light source is located 1 foot from the base along the axis of symmetry. If the opening of the searchlight is 3 feet across, find the depth.

42. An arch is in the shape of a parabola. It has a span of 100 feet and a maximum height of 20 feet. Placing the vertex at the point $(0,20)$, find the equation of the parabola and determine the height of the arch 40 feet from the center.

43. Balcony-sized solar cookers have been designed for families living in India. The top of a dish has a diameter of 1,600 mm. The sun's rays reflect off the parabolic mirror toward the "cooker", which is placed 320 mm from the base. Find the depth of the cooker.

44. The points $\left(-\frac{3}{4},\frac{1}{2}\right)$, $\left(-\frac{7}{10},1\right)$, $\left(\frac{1}{2},3\right)$, $\left(0,\frac{-\sqrt{15}+1}{2}\right)$, and $\left(0,\frac{\sqrt{15}+1}{2}\right)$ lie on the parabola $\left(y-\frac{1}{2}\right)^2=5\left(x+\frac{3}{4}\right)$. Find three other points that lie on the parabola.

5.4 Exercises

1. Define an ellipse in terms of its foci.
2. What can be said about the symmetry of the graph of an ellipse with center at the origin and foci along the *y*-axis?

In Exercises 3 – 8, write the equation in standard form if it represents an ellipse. If the equation does not represent an ellipse, explain how the equation violates the definition of an ellipse.

3. $2x^2 + y = 4$
4. $4x^2 + 9y^2 = 36$
5. $4x^2 - y^2 = 4$
6. $4x^2 + 9y^2 = 1$
7. $4x^2 - 8x + 9y^2 - 72y + 112 = 0$
8. $4x^2 + 4y^2 = 1$

In Exercises 9 – 20, find the center, the vertices, and the foci of the ellipse. Graph the ellipse.

9. $\frac{x^2}{169} + \frac{y^2}{25} = 1$
10. $\frac{x^2}{9} + \frac{y^2}{25} = 1$
11. $\frac{x^2}{25} + \frac{y^2}{36} = 1$
12. $\frac{x^2}{16} + \frac{y^2}{9} = 1$
13. $\frac{(x-2)^2}{64} + \frac{(y-4)^2}{16} = 1$
14. $\frac{x^2}{2} + \frac{(y+1)^2}{5} = 1$
15. $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$
16. $\frac{(x+5)^2}{16} + \frac{(y-4)^2}{1} = 1$
17. $\frac{(x-1)^2}{10} + \frac{(y-3)^2}{11} = 1$
18. $\frac{(x-1)^2}{9} + \frac{(y+3)^2}{4} = 1$
19. $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{20} = 1$
20. $\frac{(x-4)^2}{8} + \frac{(y-2)^2}{18} = 1$

In Exercises 21 – 30, put the equation of the ellipse in standard form. Find the center, the vertices, and the foci. Graph the ellipse.

21. $9x^2 + 25y^2 - 54x - 50y - 119 = 0$
22. $12x^2 + 3y^2 - 30y + 39 = 0$
23. $5x^2 + 18y^2 - 30x + 72y + 27 = 0$
24. $x^2 - 2x + 2y^2 - 12y + 3 = 0$
25. $9x^2 + 4y^2 - 4y - 8 = 0$
26. $6x^2 + 5y^2 - 24x + 20y + 14 = 0$
27. $4x^2 - 24x + 36y^2 - 360y + 864 = 0$
28. $4x^2 + 24x + 16y^2 - 128y + 228 = 0$
29. $4x^2 + 40x + 25y^2 - 100y + 100 = 0$
30. $9x^2 + 72x + 16y^2 + 16y + 4 = 0$

In Exercises 31 – 42, find the standard form of the equation of the ellipse that has the given properties.

31. Center $(3,7)$, Vertex $(3,2)$, Focus $(3,3)$

32. Center $(4,2)$, Vertex $(9,2)$, Focus $(4+2\sqrt{6},2)$

33. Center $(3,5)$, Vertex $(3,11)$, Focus $(3,5+4\sqrt{2})$

34. Center $(-3,4)$, Vertex $(1,4)$, Focus $(-3+2\sqrt{3},4)$

35. Foci $(0,\pm 5)$, Vertices $(0,\pm 8)$

36. Foci $(\pm 3,0)$; Segment of the minor axis within (bounded by) the ellipse has length 10.

37. Vertices $(3,2)$ and $(13,2)$; Minor axis intersects ellipse at points $(8,4)$ and $(8,0)$

38. Center $(5,2)$, Vertex $(0,2)$, eccentricity $\frac{1}{2}$

39. All points on the ellipse are in Quadrant IV except $(0,-9)$ and $(8,0)$. (One might also say that the ellipse is tangent to the axes at those two points.)

40. Center $(0,0)$, Focus $(4,0)$, containing the point $(0,3)$

41. Center $(0,0)$, Focus $(0,-2)$, containing the point $(5,0)$

42. Center $(0,0)$, Focus $(3,0)$; The segment of the major axis within (bounded by) the ellipse is twice as long as the segment of the minor axis within (bounded by) the ellipse.

In Exercises 43 – 47, given the graph of the ellipse, determine its equation.

43.

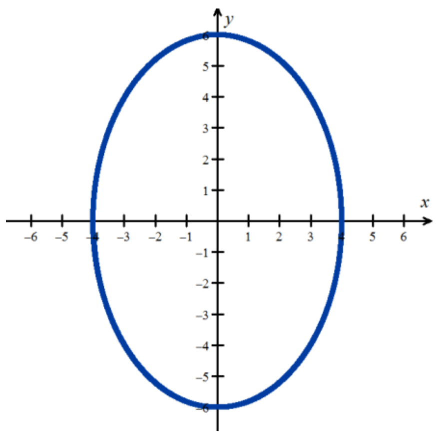

Figure Ex5.4.1

44.

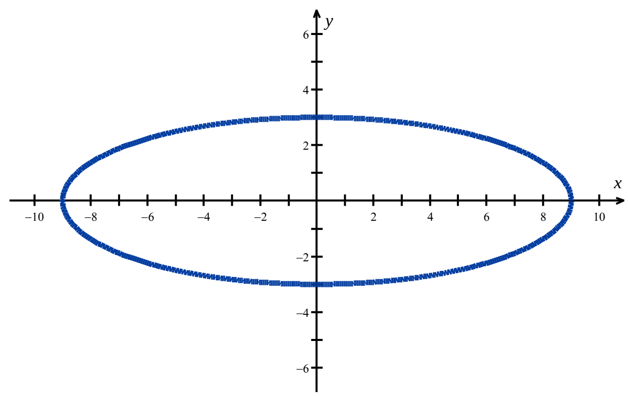

Figure Ex5.4.2

45.

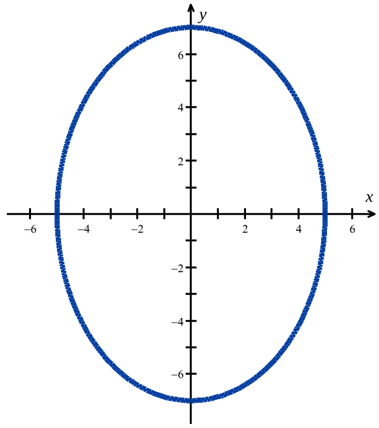

Figure Ex5.4. 3

46.

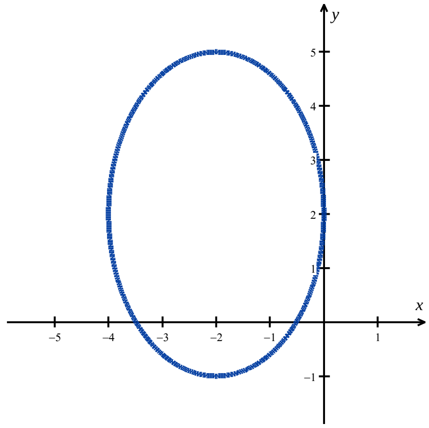

Figure Ex5.4.4

47.

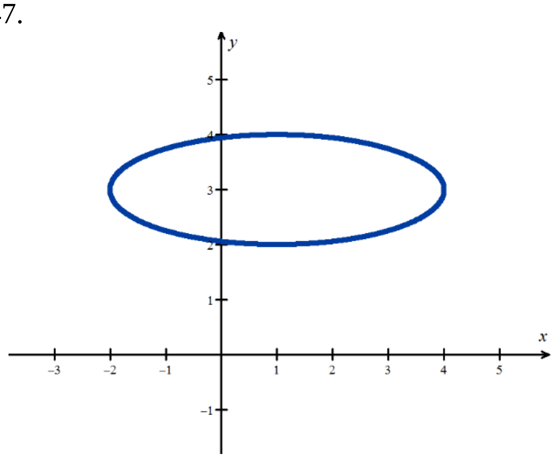

Figure Ex5.4.5

48. Find the equation of the ellipse whose vertices are $(\pm 12, 0)$ with eccentricity $e = \frac{1}{6}$.

49. Find the equation of the ellipse whose vertices are $(0, \pm 4)$ with eccentricity $e = \frac{1}{3}$.

50. An elliptical arch[12] has a height of 8 feet and a span of 20 feet. Placing the arch on a coordinate plane with highest point at $(0,8)$, find an equation for the ellipse and use it to find the height of the arch at a distance of 4 feet from the center.

51. An elliptical arch has a height of 12 feet and a span of 40 feet. Placing the arch on a coordinate plane with highest point at $(0,12)$, find an equation for the ellipse and use it to find the distance from the center to a point at which the height is 6 feet.

[12] An elliptical arch is an arch in the shape of a semi-ellipse, or the top half of an ellipse.

52. A bridge is to be built in the shape of an elliptical arch and is to have a span of 120 feet. The height of the arch at a distance of 40 feet from the center is to be 8 feet. Find the height of the arch at its center.
53. An elliptical arch is 6 feet wide at the base and 9 feet tall in the middle. Find the height of the arch exactly 1 foot in from the base of the arch. Compare your result with your answer to Exercise 39 in **Section 5.3**.
54. A person in a whispering gallery standing at one focus of the ellipse can whisper and be heard by a person standing at the other focus because all the sound waves that reach the ceiling are reflected to the other person. If a whispering gallery has a length of 120 feet, and the foci are located 30 feet from the center, find the height of the ceiling at the center.
55. A person is standing 8 feet from the nearest wall in a whispering gallery. If that person is at one focus, and the other focus is 80 feet away, what is the length of the gallery and what is its height at the center?
56. The Earth's orbit around the sun is an ellipse with the sun at one focus and eccentricity $e \approx 0.0167$. The length of the semimajor axis (that is, half of the major axis) is defined to be 1 astronomical unit (AU). The vertices of the elliptical orbit are given special names: 'aphelion' is the vertex farthest from the sun, and 'perihelion' is the vertex closest to the sun. Find the distance in AU between the sun and aphelion and the distance in AU between the sun and perihelion.
57. Some famous examples of whispering galleries include St. Paul's Cathedral in London, England, and National Statuary Hall in Washington D.C. With the help of your classmates, research these whispering galleries. How does the whispering effect compare and contrast with the scenario in **Example 5.4.7**?
58. The points $(1,-4)$, $(3,-1)$, $(5,-4)$, $(3,-7)$, $\left(\frac{-2\sqrt{5}+9}{3},-2\right)$, and $\left(\frac{2\sqrt{5}+9}{3},-2\right)$ lie on the ellipse $\frac{(x-3)^2}{4}+\frac{(y+4)^2}{9}=1$. Find three other points that lie on the ellipse.

5.5 Exercises

1. Define a hyperbola in terms of its foci.

2. What can we conclude about a hyperbola if its asymptotes intersect at the origin?

3. If the transverse axis of a hyperbola is vertical, what do we know about the graph?

In Exercises 4 – 8, write the equation in standard form if it represents a hyperbola. If the equation does not represent a hyperbola, explain how the equation violates the definition of a hyperbola.

4. $3y^2+2x=6$

5. $\dfrac{x^2}{36}-\dfrac{y^2}{9}=1$

6. $5y^2+4x^2=6x$

7. $25x^2-16y^2=400$

8. $-9x^2+18x+y^2+4y-14=0$

In Exercises 9 – 20, find the center, the vertices, and the foci of the hyperbola. Find the equations of the asymptotes and graph the hyperbola.

9. $\dfrac{x^2}{16}-\dfrac{y^2}{9}=1$

10. $\dfrac{y^2}{9}-\dfrac{x^2}{16}=1$

11. $\dfrac{x^2}{49}-\dfrac{y^2}{16}=1$

12. $\dfrac{y^2}{9}-\dfrac{x^2}{25}=1$

13. $\dfrac{(x-2)^2}{4}-\dfrac{(y+3)^2}{9}=1$

14. $\dfrac{(y-3)^2}{11}-\dfrac{(x-1)^2}{10}=1$

15. $\dfrac{(x+4)^2}{16}-\dfrac{(y-4)^2}{1}=1$

16. $\dfrac{(x+1)^2}{9}-\dfrac{(y-3)^2}{4}=1$

17. $\dfrac{(y+2)^2}{16}-\dfrac{(x-5)^2}{20}=1$

18. $\dfrac{(x-4)^2}{8}-\dfrac{(y-2)^2}{18}=1$

19. $\dfrac{(y+5)^2}{9}-\dfrac{(x-4)^2}{25}=1$

20. $\dfrac{(y-3)^2}{9}-\dfrac{(x-3)^2}{9}=1$

In Exercises 21 – 30, put the equation of the hyperbola into standard form. Find the center, the vertices, and the foci. Graph the hyperbola.

21. $12x^2-3y^2+30y-111=0$

22. $18y^2-5x^2+72y+30x-63=0$

23. $9x^2 - 25y^2 - 54x - 50y - 169 = 0$

24. $-6x^2 + 5y^2 - 24x + 40y + 26 = 0$

25. $-4x^2 + 16y^2 - 8x - 32y - 52 = 0$

26. $x^2 - 25y^2 - 8x - 100y - 109 = 0$

27. $-x^2 + 4y^2 + 8x - 40y + 88 = 0$

28. $64x^2 - 9y^2 + 128x - 72y - 656 = 0$

29. $16x^2 - 4y^2 + 64x - 8y - 4 = 0$

30. $-100x^2 + y^2 + 1000x - 10y - 2575 = 0$

In Exercises 31 – 42, find the standard form of the equation of the hyperbola that has the given properties.

31. Vertices $(0, \pm 5)$, Foci $(0, \pm 8)$

32. Vertices $(\pm 3, 0)$, Focus $(5, 0)$

33. Vertices $(0, \pm 6)$, Focus $(0, -8)$

34. Center $(0,0)$, Vertex $(0,-13)$, Focus $(0, \sqrt{313})$

35. Foci $(\pm 5, 0)$; Segment of the conjugate axis bordered by the guide rectangle has length 6.

36. Center $(3,7)$, Vertex $(3,3)$, Focus $(3,2)$

37. Center $(4,2)$, Vertex $(9,2)$, Focus $(4+\sqrt{26}, 2)$

38. Center $(3,5)$, Vertex $(3,11)$, Focus $(3, 5+2\sqrt{10})$

39. Vertices $(0,1)$ and $(8,1)$, Focus $(-3,1)$

40. Vertices $(1,1)$ and $(11,1)$, Focus $(12,1)$

41. Vertices $(3,2)$ and $(13,2)$; Guide rectangle intersects conjugate axis at $(8,4)$ and $(8,0)$.

42. Vertex $(-10,5)$, Asymptotes $y = \pm \frac{1}{2}(x-6)+5$

In Exercises 43 – 47, given the graph of the hyperbola, determine its equation.

43.

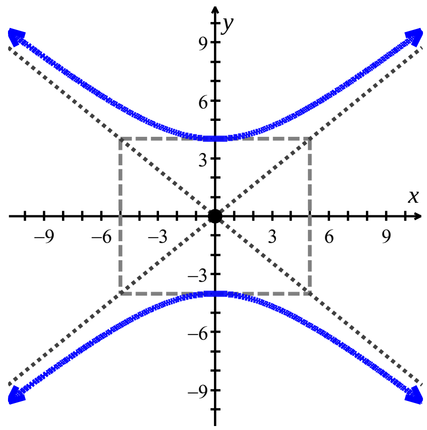

Figure Ex5.5. 1

44.

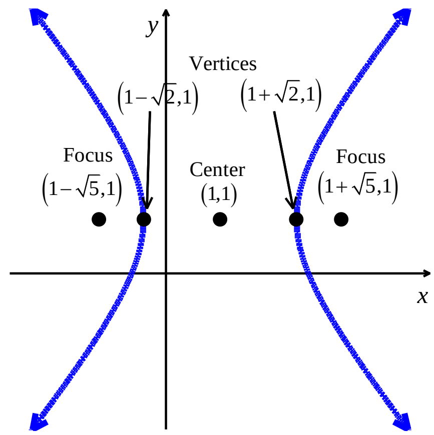

Figure Ex5.5. 2

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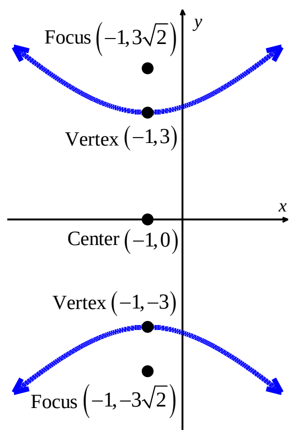

Figure Ex5.5. 3

46.

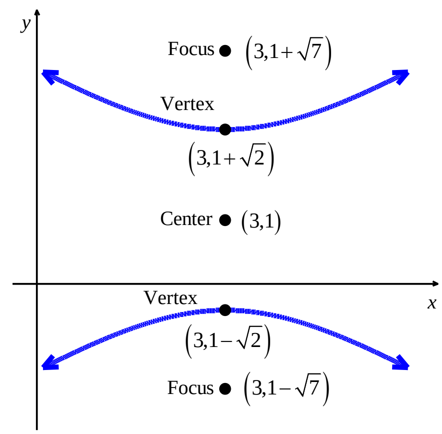

Figure Ex5.5. 4

47.

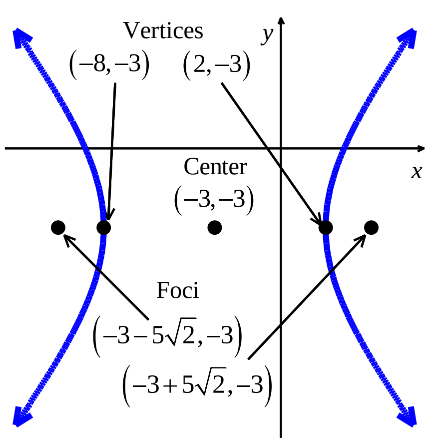

Figure Ex5.5. 5

48. The design layout of a hyperbolic cooling tower is shown below. The tower stands 167.082 meters tall. The diameter of the top is 60 meters. At their closest, the sides of the tower are 40 meters apart. Find the equation of the hyperbola that models the sides of the cooling tower. Assume that the center of the hyperbola is the origin of the coordinate plane. Round final values to four decimal places.

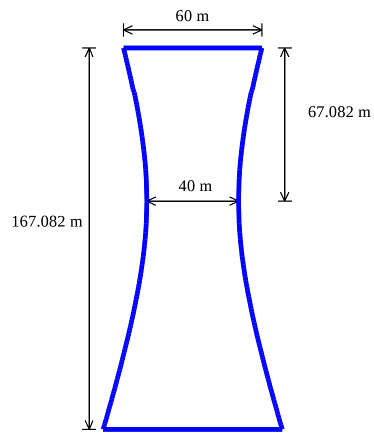

Figure Ex5.5. 6

49. The cross section of a hyperbolic cooling tower is shown below. Suppose the tower is 450 feet wide at the base, 275 feet wide at the top, and 220 feet at its narrowest point (which occurs 330 feet above the ground). Determine the height of the tower to the nearest foot.

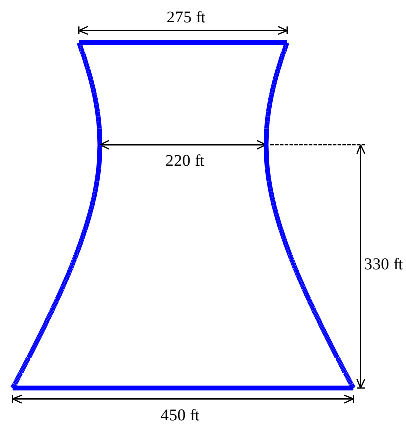

Figure Ex5.5. 7

50. A hedge is to be constructed in the shape of a hyperbola near a fountain at the center of a yard. The hedge will follow the asymptotes $y = x$ and $y = -x$, and its closest distance to the center fountain is 5 yards. Find the equation of the hyperbola and sketch the graph.

51. A hedge is to be constructed in the shape of a hyperbola near a fountain at the center of a yard. The hedge will follow the asymptotes $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$, and its closest distance to the center fountain is 20 yards. Find the equation of the hyperbola and sketch the graph.

52. With the help of your classmates, show that if $Ax^2 + By^2 + Cx + Dy + E = 0$ determines a non-degenerate conic[15] then

- $AB < 0$ means that the graph is a hyperbola
- $AB = 0$ means that the graph is a parabola
- $AB > 0$ means that the graph is an ellipse or circle

In Exercises 53 – 62, find the standard form of the equation using the Strategies for Identifying Conic Sections and then graph any resulting conic sections.

53. $x^2 - 2x - 4y - 11 = 0$

54. $x^2 + y^2 - 8x + 4y + 11 = 0$

55. $9x^2 + 4y^2 - 36x + 24y + 36 = 0$

56. $9x^2 - 4y^2 - 36x - 24y - 36 = 0$

57. $y^2 + 8y - 4x + 16 = 0$

58. $4x^2 + y^2 - 8x + 4 = 0$

59. $4x^2 + 9y^2 - 8x + 54y + 49 = 0$

60. $x^2 + y^2 - 6x + 4y + 14 = 0$

61. $2x^2 + 4y^2 + 12x - 8y + 25 = 0$

62. $4x^2 - 5y^2 - 40x - 20y + 160 = 0$

63. The points $(-4,2)$, $(2,2)$, $\left(\frac{-3\sqrt{29}-5}{5}, 0\right)$, $\left(\frac{3\sqrt{29}-5}{5}, 0\right)$, $\left(4, -\frac{14}{3}\right)$ and $\left(4, \frac{26}{3}\right)$ lie on the hyperbola $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{25} = 1$. Find three other points that lie on the hyperbola.

[15] Recall that this means its graph is either a circle, parabola, ellipse or hyperbola.