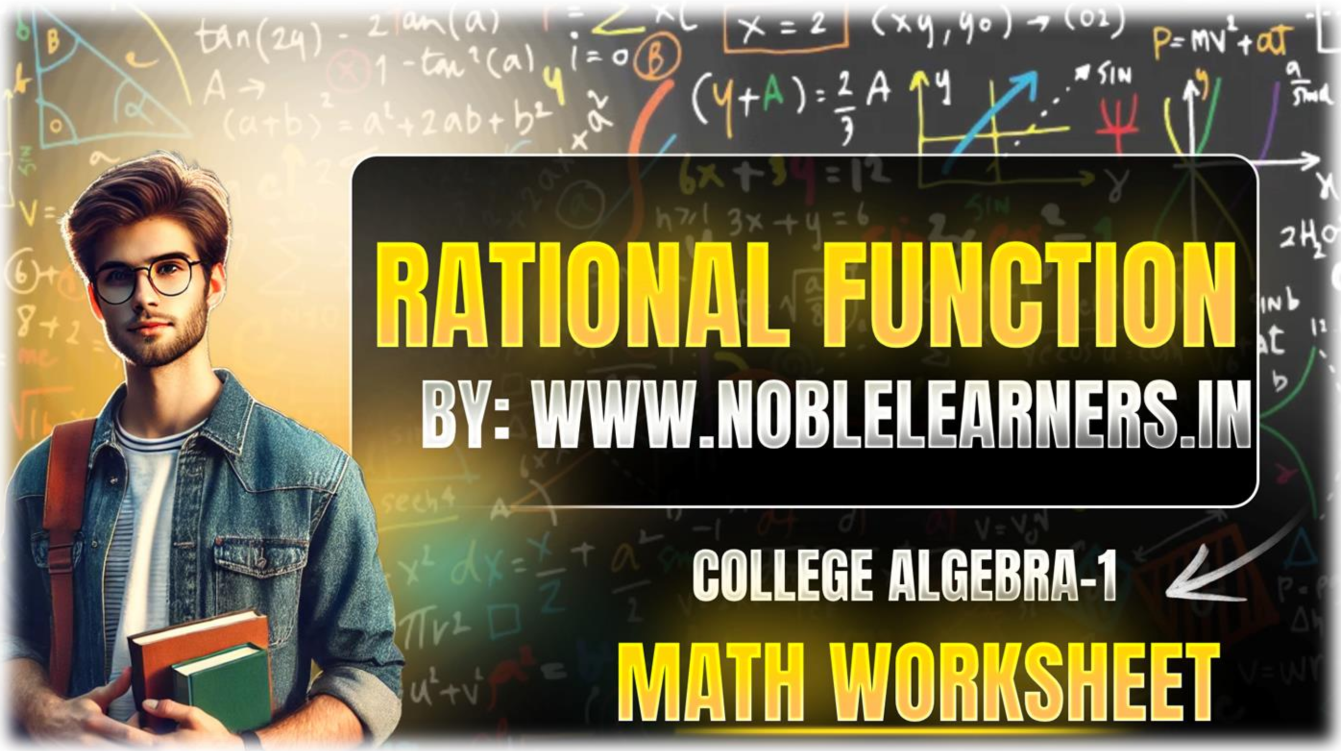

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3.1 Exercises

1. Can the graph of a rational function have no vertical asymptotes? Explain.

2. Can the graph of a rational function have no *x*-intercepts? Explain.

In Exercises 3 – 6, determine if the given function is a rational function. Explain your reasoning.

3. $f(x)=\dfrac{6}{x}$

4. $f(x)=\sqrt{x}-6$

5. $f(x)=\dfrac{1-3x^2}{4x^{\pi}+x^2-1}$

6. $f(x)=\dfrac{2x^6-30}{x^2+5x+3}$

In Exercises 7 – 12, find the domain of the rational function.

7. $f(x)=\dfrac{x-1}{x+2}$

8. $f(x)=\dfrac{x+1}{x^2-1}$

9. $f(x)=\dfrac{x^2+4}{x^2-2x-8}$

10. $f(x)=\dfrac{x^2+4x-3}{x^4-5x^2+4}$

11. $f(x)=\dfrac{x+1}{x^2+25}$

12. $f(x)=\dfrac{x+5}{x^2+25}$

In Exercises 13 – 18, find the *x*- and *y*-intercepts for the rational function.

13. $f(x)=\dfrac{x+5}{x^2+4}$

14. $f(x)=\dfrac{x}{x^2-x}$

15. $f(x)=\dfrac{x^2+8x+7}{x^2+11x+30}$

16. $f(x)=\dfrac{x^2+x+6}{x^2-10x+24}$

17. $f(x)=\dfrac{94-2x^2}{3x^2-12}$

18. $f(x)=\dfrac{x+5}{x^2-25}$

In Exercises 19 – 24, state the vertical asymptote(s). Then determine the end behavior for the rational function and give your answer by stating the following and filling in the blanks.

As $x\to-\infty$, $f(x)\to$ ________, and as $x\to\infty$, $f(x)\to$ ________.

19. $f(x)=\dfrac{4}{x-1}$

20. $f(x)=\dfrac{x-4}{x-6}$

21. $f(x)=\dfrac{x}{x^2-9}$

22. $f(x)=\dfrac{x}{x^2+5x-36}$

23. $f(x)=\dfrac{3x^2+2}{4x^2-1}$

24. $f(x)=\dfrac{3x-4}{x^3-16x}$

In Exercises 25 – 30, identify and state the vertical and horizontal asymptotes for the rational function.

25. $f(x)=\dfrac{x^2-1}{x^3+9x^2+14x}$

26. $f(x)=\dfrac{x+5}{x^2+25}$

27. $f(x)=\dfrac{2}{5x+2}$

28. $f(x)=\dfrac{3+x}{x^3-27}$

29. $f(x)=\dfrac{4-2x}{3x-1}$

30. $f(x)=\dfrac{2x^2-8}{2x^2-4x+2}$

In Exercises 31 – 38, state the domain, intercepts, and asymptotes. Graph the rational function, drawing asymptotes as dashed lines.

31. $f(x)=\dfrac{1}{x-2}$

32. $f(x)=\dfrac{4}{x+2}$

33. $f(x)=\dfrac{2x-3}{x+4}$

34. $f(x)=\dfrac{x-5}{3x-1}$

35. $f(x)=\dfrac{x-3}{x}$

36. $f(x)=\dfrac{5x}{6-2x}$

37. $f(x)=\dfrac{x}{3x-6}$

38. $f(x)=\dfrac{3+7x}{5-2x}$

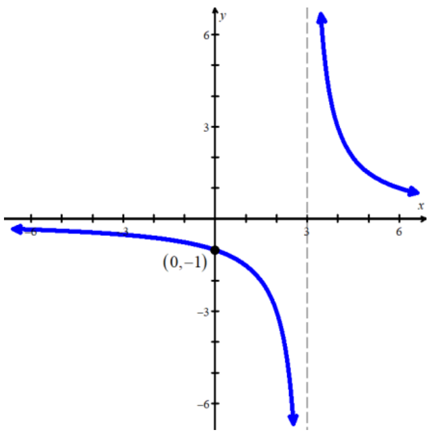
39. Find an equation for a rational function, $r(x)$, that has an x-intercept $(1,0)$, a vertical asymptote $x=4$, and a horizontal asymptote $y=3$. The degree of both the numerator and the denominator of $r(x)$ is 1.

40. Find an equation for a rational function, $r(x)$, that has an x-intercept $(-2,0)$, a vertical asymptote $x=0$, and a horizontal asymptote $y=-\dfrac{3}{7}$. The degree of both the numerator and the denominator of $r(x)$ is 1.

In Exercises 41 – 44, find an equation for a rational function, $r(x)$, that has the given graph. Write your function in the form $r(x)=\dfrac{ax+b}{cx+d}$.

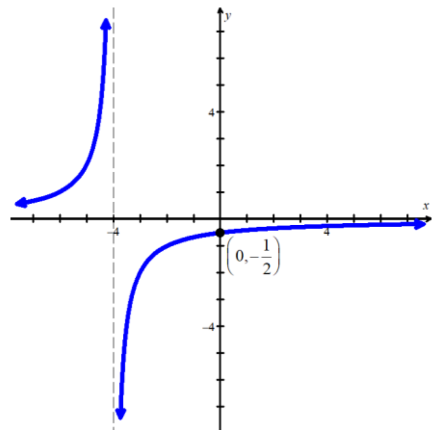
41.

Figure Ex3.1. 1



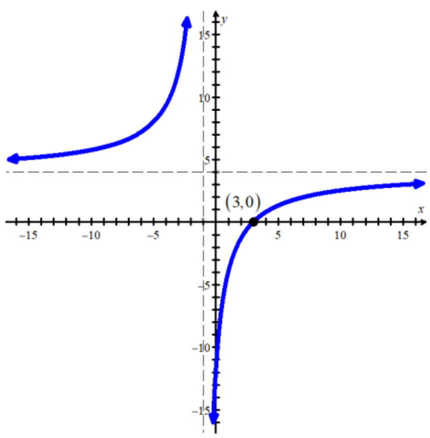
42.

Figure Ex3.1. 2



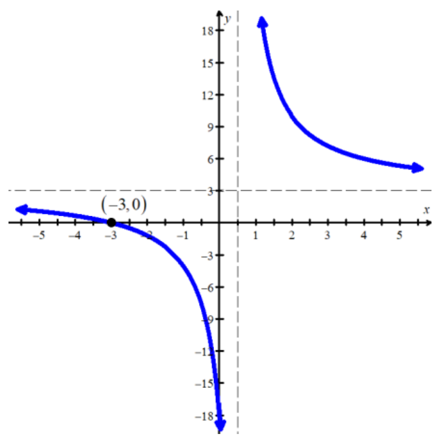
43.

Figure Ex3.1. 3



44.

Figure Ex3.1. 4



45. The cost C in dollars to remove $p\%$ of the invasive species of Ippizuti fish from Sasquatch Pond is given by $C(p)=\dfrac{1770p}{100-p}$, $0\le p<100$.

(a) Find and interpret $C(25)$ and $C(95)$.

(b) What does the vertical asymptote at $x=100$ mean within the context of the problem?

(c) What percentage of the Ippizuti fish can you remove for $40,000?

46. The population of Sasquatch in Portage County can be modeled by the function $P(t)=\dfrac{150t}{t+15}$ where $t=0$ represents the year 1803. Find the horizontal asymptote of the graph of $y=P(t)$ and explain what it means.

3.2 Exercises

1. How can multiplicities be used in graphing rational functions?

2. What property of a rational function results in its graph crossing the *x*-axis at an intercept?

In Exercises 3 – 14, state the domain, intercepts, and asymptotes. Graph the rational function, drawing asymptotes as dashed lines.

3. $f(x)=\dfrac{1}{x^2}$

4. $f(x)=\dfrac{1}{x^2+x-12}$

5. $f(x)=\dfrac{x}{x^2+x-12}$

6. $f(x)=\dfrac{4x}{x^2-4}$

7. $f(x)=\dfrac{3x^2-5x-2}{x^2-9}$

8. $f(x)=\dfrac{x+7}{x^2+6x+9}$

9. $f(x)=\dfrac{4}{x^2-4x+4}$

10. $f(x)=\dfrac{5}{x^2+2x+1}$

11. $f(x)=\dfrac{3x^2-14x-5}{3x^2+8x-16}$

12. $f(x)=\dfrac{2x^2+7x-15}{3x^2-14x+15}$

13. $f(x)=\dfrac{(x-1)(x+3)(x-5)}{(x+2)^2(x-4)}$

14. $f(x)=\dfrac{(x+2)^2(x-5)}{(x-3)(x+1)(x+4)}$

In Exercises 15 – 20, write an equation for a rational function with the given characteristics.

15. Vertical asymptotes at $x=5$ and $x=-5$; *x*-intercepts at $(2,0)$ and $(-1,0)$; *y*-intercept at $(0,4)$

16. Vertical asymptotes at $x=-4$ and $x=-1$; *x*-intercepts at $(1,0)$ and $(5,0)$; *y*-intercept at $(0,7)$

17. Vertical asymptotes at $x=-4$ and $x=-5$; *x*-intercepts at $(4,0)$ and $(-6,0)$; horizontal asymptote at $y=7$

18. Vertical asymptotes at $x=-3$ and $x=6$; *x*-intercepts at $(-2,0)$ and $(1,0)$; horizontal asymptote at $y=-2$

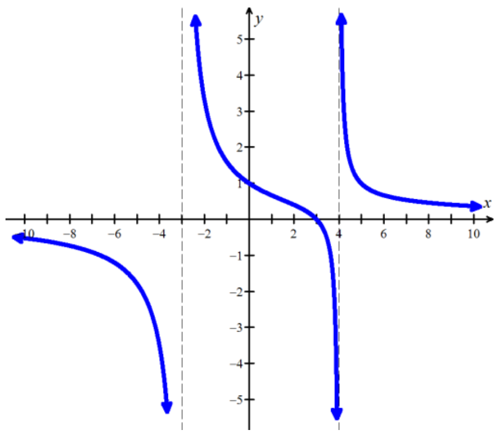
19. Vertical asymptote at $x=-1$; graph touches but does not cross *x*-axis at $(2,0)$; *y*-intercept at $(0,2)$

20. Vertical asymptote at $x=3$; graph touches but does not cross *x*-axis at $(1,0)$; *y*-intercept at $(0,4)$

In Exercises 21 – 26, use the graph to write an equation for the function.

21.

Figure Ex3.2. 1



22.

Figure Ex3.2. 2

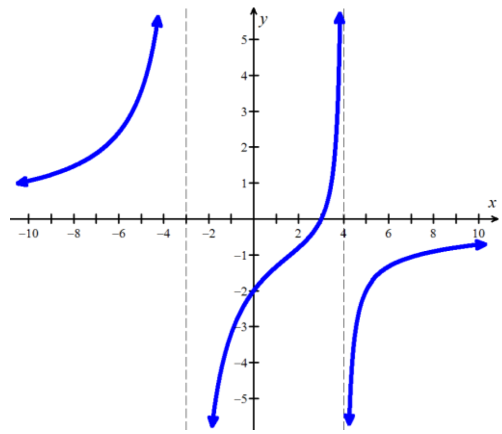


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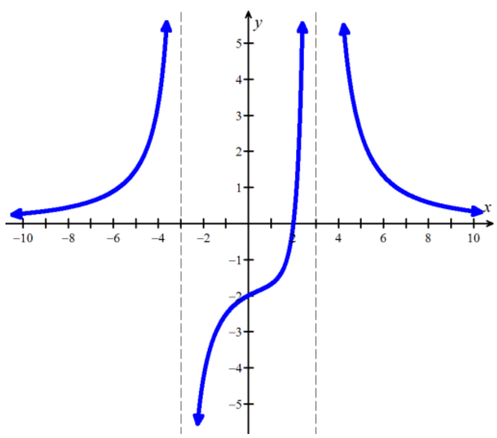
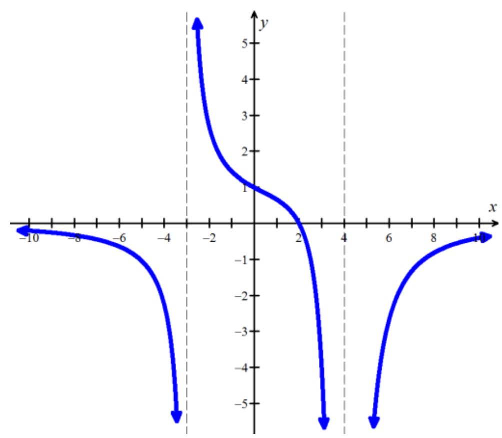
Figure Ex3.2. 3



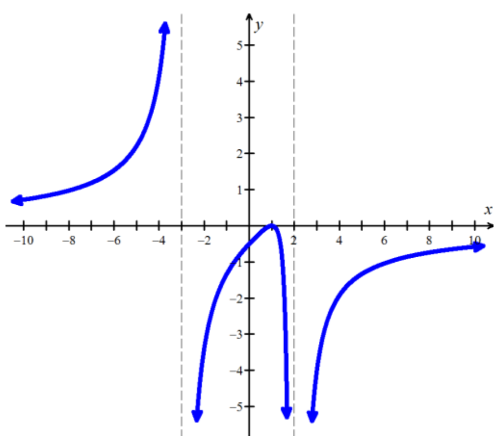
24.

Figure Ex3.2. 4



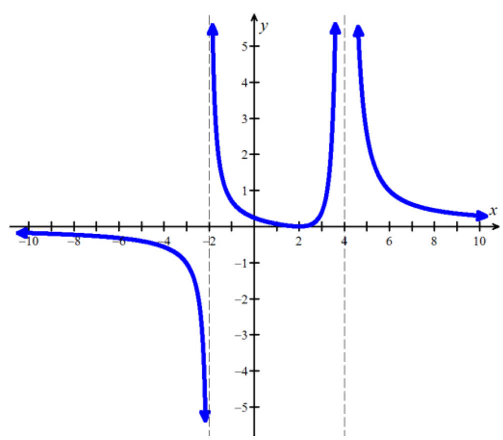
25.

Figure Ex3.2. 5



26.

Figure Ex3.2. 6



3.3 Exercises

1. What characteristics of a rational function indicate the absence of vertical asymptotes?

2. What characteristics of a rational function indicate the presence of a slant asymptote?

In Exercises 3 – 8, determine if a slant asymptote exists. If a slant asymptote exists, state its equation.

3. $f(x)=\dfrac{24x^2+6x}{2x+1}$

4. $f(x)=\dfrac{4x^2-10}{2x-4}$

5. $r(x)=\dfrac{81x^2-18}{3x-2}$

6. $f(x)=\dfrac{6x^3-5x}{3x^2+4}$

7. $f(x)=\dfrac{x^3}{1-x}$

8. $g(x)=\dfrac{x^2+5x+4}{x-1}$

In Exercises 9 – 12, state the holes in the graph of the rational function as ordered pairs. Write the rational function in its reduced form with the domain restriction.

9. $f(x)=\dfrac{x^2+2x-3}{x^2-1}$

10. $r(x)=\dfrac{x^2-x-6}{x^2-4}$

11. $r(x)=\dfrac{2x-1}{-2x^2-5x+3}$

12. $f(x)=\dfrac{x^2-x-12}{x^2+x-6}$

In Exercises 13 – 32, state the domain, location of any holes (written as ordered pairs), intercepts, and asymptotes. Graph the rational function, drawing asymptotes as dashed lines.

13. $f(x)=\dfrac{x^2+2x-3}{x^2-1}$

14. $r(x)=\dfrac{x^2-x-6}{x^2-4}$

15. $f(x)=\dfrac{2x^2+x-1}{x-4}$

16. $g(x)=\dfrac{2x^2-3x-20}{x-5}$

17. $r(x)=\dfrac{2x-1}{-2x^2-5x+3}$

18. $r(x)=\dfrac{4x}{x^2+4}$

19. $g(x)=\dfrac{x^2-x-12}{x^2+x-6}$

20. $f(x)=\dfrac{x^2-x-6}{x+1}$

21. $f(x)=\dfrac{x^2-x}{3-x}$

22. $h(x)=\dfrac{x^3+2x^2+x}{x^2-x-2}$

23. $r(x)=\dfrac{x^2-2x+1}{x^3+x^2-2x}$

24. $f(x)=\dfrac{x^2-1}{x^2-2x-3}$

25. $r(x)=\dfrac{x-2}{x^2-4}$

26. $f(x)=\dfrac{x^2-25}{x^3-6x^2+5x}$

27. $f(x)=\dfrac{x^3+1}{x^2-1}$

28. $f(x)=\dfrac{x^3-3x+1}{x^2+1}$

29. $r(x)=\dfrac{-x^3+4x}{x^2-9}$

30. $g(x)=\dfrac{18-2x^2}{x^2-9}$

31. $f(x)=\dfrac{2x^3-x^2-3x}{x^3-6x^2+9x}$

32. $r(x)=\dfrac{3(x-2)^2(x+3)}{(2x-3)^2(x+1)^3}$

33. The six-step graphing procedure outlined in **Section 3.3** cannot tell us everything of importance about the graph of a rational function. Without Calculus, we may use graphing technology to reveal the hidden mysteries of rational function behavior. Working with your classmates, use graphing technology to examine the graphs of the following rational functions. Compare and contrast their features. Which of the features can the six-step process reveal and which features cannot be detected by it?

(a) $f(x)=\dfrac{1}{x^2+1}$

(b) $g(x)=\dfrac{x}{x^2+1}$

(c) $h(x)=\dfrac{x^2}{x^2+1}$

(d) $r(x)=\dfrac{x^3}{x^2+1}$

3.4 Exercises

1. Give an example of a rational equation that does not have a solution.

2. Give an example of a rational inequality that has a single solution.

In Exercises 3 – 8, solve the rational equation. Check domains to eliminate extraneous solutions.

3. $\frac{2x-3}{x+4}=0$

4. $\frac{2x-3}{x+4}=5$

5. $\frac{x}{5x+4}=3$

6. $\frac{3x-1}{x^2+1}=1$

7. $\frac{2x+17}{x+1}=x+5$

8. $\frac{x^2-2x+1}{x^3+x^2-2x}=1$

In Exercises 9 – 34, solve the rational inequality. Express your answer using interval notation.

9. $\frac{2}{x+1}>0$

10. $\frac{1}{x+2}\geq 0$

11. $\frac{4}{2x-3}\leq 0$

12. $\frac{2}{(x-1)(x+2)}<0$

13. $\frac{x-3}{x+2}\leq 0$

14. $\frac{x+2}{(x-1)(x-4)}\geq 0$

15. $\frac{(x+3)^2}{(x-1)^2(x+1)}>0$

16. $\frac{x}{x^2-1}>0$

17. $\frac{4x}{x^2+4}\geq 0$

18. $\frac{x^2-x-12}{x^2+x-6}>0$

19. $\frac{3x^2-5x-2}{x^2-9}<0$

20. $\frac{x^3+2x^2+x}{x^2-x-2}\geq 0$

21. $\frac{x^2+5x+6}{x^2-1}>0$

22. $\frac{3x-1}{x^2+1}\leq 1$

23. $\frac{2x-1}{x+3}\leq -5$

24. $\frac{3x+5}{x-4}<2$

25. $\frac{x+5}{x-3}\leq -4$

26. $\frac{3x+1}{x-2}>5$

27. $\dfrac{2x+17}{x+1} > x+5$

28. $\dfrac{1}{x^2+1} < 0$

29. $\dfrac{x^4-4x^3+x^2-2x-15}{x^3-4x^2} \ge x$

30. $\dfrac{5x^3-12x^2+9x+10}{x^2-1} \ge 3x-1$

31. $\dfrac{6}{x^2-x-2} \ge \dfrac{x+3}{x^2-1}$

32. $\dfrac{1}{x-3}+\dfrac{1}{x+3} < \dfrac{10}{x^2-9}$

33. $\dfrac{1}{x^2-x-6}+\dfrac{2}{x^2+2x} \le \dfrac{15}{x^2-3x}$

34. $\dfrac{6}{x+1}+\dfrac{x^2-5x}{x^3+x^2-x-1} \ge 1+\dfrac{5}{x^2+2x+1}$

35. Given $f(x)=\sqrt{2-4x}$ and $g(x)=-\dfrac{3}{x}$, find the domain of the composite function $(f \circ g)(x)$.

36. The population of Sasquatch in Salt Lake County was modeled by the function $P(t)=\dfrac{150t}{t+15}$, where $t=0$ represents the year 1803. According to this model, when were there fewer than 100 Sasquatch in Salt Lake County?

37. Following surgery, a patient has been receiving a pain relief medication intravaniously. The concentration C (in milligrams per liter) of the medication in the patient's bloodstream t hours after this process started is given by $C(t)=\dfrac{50t+10}{t+2}$, $t \ge 0$.

a) The patient will not receive pain relief unless the concentration of the medication is 20 or more milligrams per liter. Use this function to determine the time interval for which the concentration of the medication, C, will be greater than or equal to 20 milligrams per liter.

b) As time goes on (as $t \to \infty$), what value will the concentration of the medication approach?

38. At a manufacturing plant that assembles clock radios, each of the assembly workers is responsible for assembling an entire radio from start to finish. Due to turnover in the workforce, new assembly workers are being hired on a regular basis. Based on past performance, the manager of the plant has determined that the number of radios, N, assembled by a worker each week after t weeks of working at the plant is given by $N(t)=\dfrac{45t+6}{3t+2}$, $t \ge 0$.

a) When an employee is able to assemble $N=12$ radios in a week, their probation period is ended and they are given a pay raise. According to this model, at what time t will the new employee be able to assemble 12 radios in a week?

b) According to this model, what is the limiting value for the number of radios an employee can assemble in a week as time increases, that is, as $t \to \infty$?