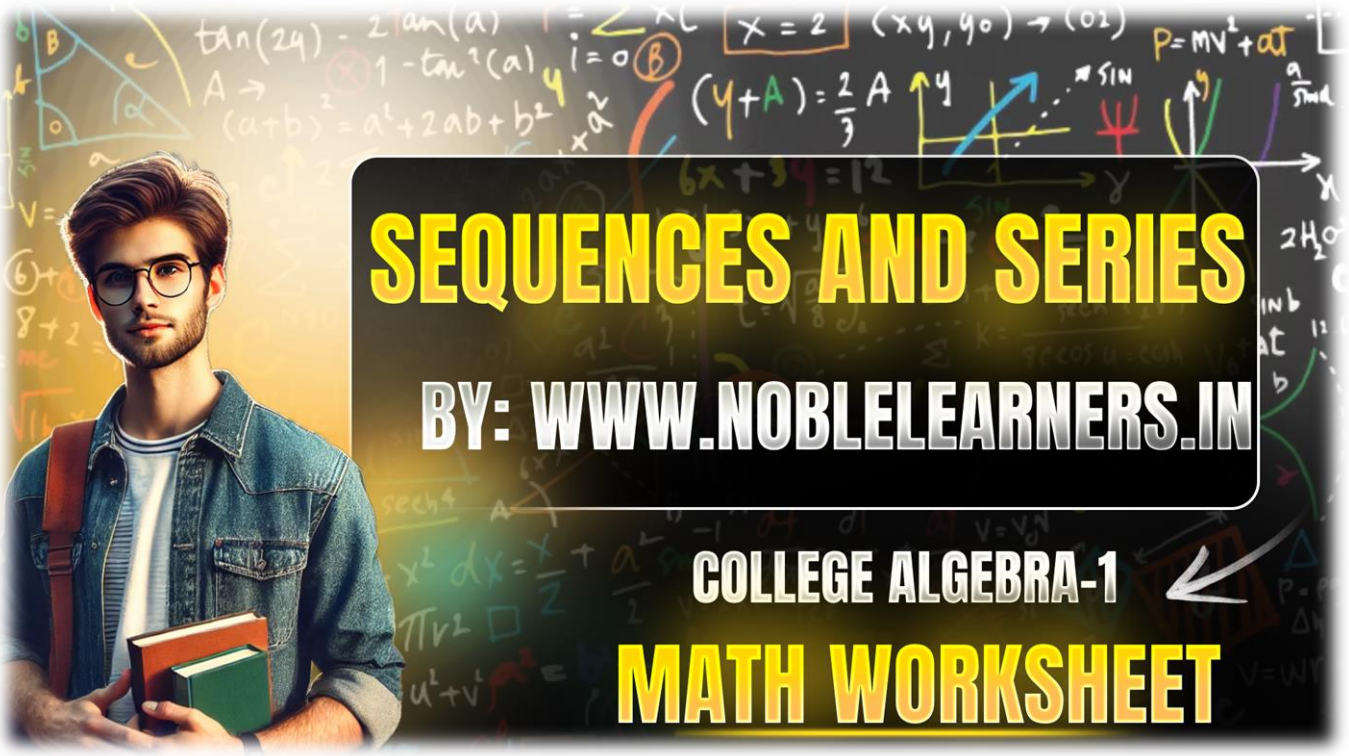

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7.1 Exercises

1. What are the main differences between using a recursive formula and using an explicit formula to describe an arithmetic sequence?

2. Describe the similarities between exponential functions and geometric sequences. How are they different?

In Exercises 3 – 12, describe how subsequent terms may be determined and state the next two terms in the sequence.

3. $3,\ 5,\ 7,\ 9,\ldots$

4. $\frac{1}{16},-\frac{1}{8},\ \frac{1}{4},-\frac{1}{2},\ldots$

5. $1,\ \frac{2}{3},\ \frac{4}{5},\ \frac{8}{7},\ldots$

6. $1,\ \frac{2}{3},\ \frac{1}{3},\ \frac{4}{27},\ldots$

7. $1,\ \frac{1}{4},\ \frac{1}{9},\ \frac{1}{16},\ldots$

8. $4,\ 7,\ 12,\ 19,\ 28,\ldots$

9. $-4,\ 2,-10,\ 14,-34,\ldots$

10. $1,\ 1,\ \frac{4}{3},\ 2,\ \frac{16}{5},\ldots$

11. $-\frac{1}{2},\ \frac{1}{4},-\frac{1}{8},\ \frac{1}{16},\ldots$

12. $1,\ 2,\ 6,\ 24,\ 120,\ldots$

In Exercises 13 – 22, find the first four terms of each sequence. Assume n is a natural number, $n\geq 2$.

13. $a_1=9,\ a_n=a_{n-1}+n$

14. $a_1=3,\ a_n=(-3)a_{n-1}$

15. $a_1=-4,\ a_n=\frac{a_{n-1}+2n}{a_{n-1}-1}$

16. $a_1=-1,\ a_n=\frac{(-3)^{n-1}}{a_{n-1}-2}$

17. $a_1=-30,\ a_n=(2+a_{n-1})\left(\frac{1}{2}\right)^n$

18. $a_1=3,\ a_n=a_{n-1}-1$

19. $a_1=12,\ a_n=\frac{a_{n-1}}{100}$

20. $a_1=2,\ a_n=3a_{n-1}+1$

21. $a_1=-2,\ a_n=\frac{a_{n-1}}{(n+1)(n+2)}$

22. $a_1=117,\ a_n=\frac{1}{a_{n-1}}$

In Exercises 23 - 24, find the first four terms of each sequence. Assume n is a natural number, $n\geq 3$.

23. $a_1=\frac{1}{24},\ a_2=1,\ a_n=(2a_{n-2})(3a_{n-1})$

24. $a_1=-1,\ a_2=5,\ a_n=a_{n-2}(3-a_{n-1})$

In Exercises 25 – 36, determine if the sequence is arithmetic, geometric or neither. If it is arithmetic, find the common difference d ; if it is geometric, find the common ratio r . Plot the first 5 terms of the sequence.

25. $-6,-12,-24,-48,-96,\ldots$

26. $11.4,\ 9.3,\ 7.2,\ 5.1,\ 3,\ldots$

27. $\frac{1}{3},\ \frac{1}{6},\ \frac{1}{12},\ \frac{1}{24},\ldots$

28. $-1,\ \frac{1}{2},-\frac{1}{4},\ \frac{1}{8},-\frac{1}{16},\ldots$

29. $17,\ 5,-7,-19,\ldots$

30. $4,\ 16,\ 64,\ 256,\ 1024,\ldots$

31. $6,\ 8,\ 11,\ 15,\ 20,\ldots$

32. $2,\ 22,\ 222,\ 2222,\ldots$

33. $0.9,\ 9,\ 90,\ 900,\ldots$

34. $\{3n-5\}_{n=1}^{\infty}$

35. $a_n = n^2 + 3n + 2,\ n \geq 1$

36. $\left\{3\left(\frac{1}{5}\right)^{n-1}\right\}_{n=1}^{\infty}$

In Exercises 37 – 45, find an explicit formula for the n th term of the sequence.

37. $3,\ 5,\ 7,\ 9,\ldots$

38. $32,\ 24,\ 16,\ 8,\ldots$

39. $-2,-4,-8,-16,\ldots$

40. $1,\ 3,\ 9,\ 27,\ldots$

41. $-5,\ 95,\ 195,\ 295,\ldots$

42. $-17,-217,-417,-617,\ldots$

43. $-1,-\frac{4}{5},-\frac{16}{25},-\frac{64}{125},\ldots$

44. $2,\ \frac{1}{3},\ \frac{1}{18},\ \frac{1}{108},\ldots$

45. $3,-1,\ \frac{1}{3},-\frac{1}{9},\ldots$

In Exercises 46 – 49, use the given information to write the first five terms of the arithmetic sequence.

46. $a_1 = -25,\ d = -9$

47. $a_1 = 0,\ d = \frac{2}{3}$

48. $a_1 = 17,\ a_7 = -31$

49. $a_{13} = -60,\ a_{33} = -160$

In Exercises 50 – 53, use the given information to write the first five terms of the geometric sequence.

50. $a_1 = 8,\ r = 0.3$

51. $a_1 = 5,\ r = \frac{1}{5}$

52. $a_7 = 64,\ a_{10} = 512$

53. $a_6 = 25,\ a_9 = -3.125$

In Exercises 54 – 59, use the given information to find the specified term of the arithmetic sequence.

54. Find a_5 if $a_1 = 3$ and $d = 4$

55. Find a_6 if $a_1 = 6$ and $d = 7$

56. Find a_1 if $a_6 = 12$ and $a_{14} = 28$

57. Find a_1 if $a_7 = 21$ and $a_{15} = 42$

58. Find a_4 if $a_1 = 33$ and $a_7 = -15$

59. Find a_{21} if $a_3 = -17.1$ and $a_{10} = -15.7$

In Exercises 60 – 65, use the given information to find the specified term of the geometric sequence.

60. Find a_5 if $a_1 = 2$ and $r = 3$

61. Find a_4 if $a_1 = 16$ and $r = -\frac{1}{3}$

62. Find a_{12} in the sequence $-1,\ 2, -4,\ 8, \ldots$

63. Find a_7 in the sequence $-2,\ \frac{2}{3}, -\frac{2}{9},\ \frac{2}{27}, \ldots$

64. Find a_8 if $a_1 = 4$ and $a_n = -3a_{n-1}$

65. Find a_{12} if $a_n = -\left(-\frac{1}{3}\right)^{n-1}$

66. Which term of the arithmetic sequence $1, 5, 9, 13, \ldots$ is 185? In other words, if $a_1 = 1$, $a_2 = 5$, and so forth, what is the value of n for which $a_n = 185$?

67. Which term of the arithmetic sequence $47, 44, 41, \ldots$ is -556? In other words, if $a_1 = 47$, $a_2 = 44$, and so forth, what is the value of n for which $a_n = -556$?

68. Which term of the geometric sequence $12, 6, 3, \ldots$ is $\frac{3}{64}$? In other words, if $a_1 = 12$, $a_2 = 6$, and so forth, what is the value of n for which $a_n = \frac{3}{64}$?

69. Find a formula for the general term a_n of the sequence $\left\{-\frac{11}{3}, \frac{13}{9}, -\frac{15}{27}, \frac{17}{81}, -\frac{19}{243}, \ldots\right\}$, assuming the pattern of the first few terms continues, and that the first term is $a_1 = -\frac{11}{3}$.

7.2 Exercises

1. What is the difference between an arithmetic sequence and an arithmetic series?

2. Describe the criteria for determining if an infinite geometric series has a finite sum. Give an example of an infinite geometric series that has a finite sum, and another that does not.

In Exercises 3 – 11, find the value of the sum.

3. $\sum_{a=1}^{14} a$

4. $\sum_{n=1}^{6} n(n-2)$

5. $\sum_{k=1}^{17} k^2$

6. $\sum_{g=4}^{9} (5g+3)$

7. $\sum_{k=3}^{8} \frac{1}{k}$

8. $\sum_{j=0}^{5} 2^j$

9. $\sum_{k=0}^{2} (3k-5)x^k$

10. $\sum_{i=1}^{4} \frac{1}{4}(i^2+1)$

11. $\sum_{n=1}^{100} (-1)^n$

In Exercises 12 – 21, rewrite the sum using summation notation.

12. $8+11+14+17+20$

13. $1-2+3-4+5-6+7-8$

14. $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}$

15. $1+2+4+\cdots+2^{29}$

16. $2+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}$

17. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{179\cdot 180}$

18. $7+4+1-2-\cdots-83$

19. $-\ln(3)+\ln(4)-\ln(5)+\cdots+\ln(20)$

20. $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\frac{1}{25}-\frac{1}{36}$

21. $\frac{1}{2}(x-5)+\frac{1}{4}(x-5)^2+\frac{1}{6}(x-5)^3+\frac{1}{8}(x-5)^4$

In Exercises 22 – 41, use formulas from this section to find the sum, if possible.

22. $\sum_{n=1}^{10} (5n+3)$

23. $\sum_{n=1}^{20} (2n-1)$

24. $\sum_{k=0}^{15} (3-k)$

25. $\sum_{n=1}^{10} \left(\frac{1}{2}\right)^n$

26. $\sum_{n=1}^{5} \left(\frac{3}{2}\right)^n$

27. $\sum_{k=0}^{5} 2\left(\frac{1}{4}\right)^k$

28. $1+4+7+\cdots+295$

29. $4+2+0-2-\cdots-146$

30. $1+3+9+\cdots+2187$

31. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{256}$

32. $3-\frac{3}{2}+\frac{3}{4}-\frac{3}{8}+\cdots+\frac{3}{256}$

33. $4+2+1+\frac{1}{2}+\cdots$

34. $\frac{1}{2}+1+2+4+\cdots$

35. $-1-\frac{1}{4}-\frac{1}{16}-\frac{1}{64}-\cdots$

36. $\sum_{k=1}^{\infty}3\left(\frac{1}{4}\right)^{k-1}$

37. $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n}$

38. $1-\frac{3}{4}+\frac{9}{16}-\frac{27}{64}+\cdots$

39. $\sum_{n=1}^{\infty}4\left(-\frac{1}{2}\right)^{n-1}$

40. $\sum_{n=1}^{\infty}8\left(\frac{4}{5}\right)^{n-1}$

41. $\sum_{j=1}^{\infty}\frac{3}{7}(4)^{j-1}$

42. The sum of terms $50-k^2$ from $k=x$ through $k=7$ is 115. What is x?

43. Write an explicit formula for a_k such that $\sum_{k=0}^{6}a_k=189$. Assume this is an arithmetic series.

44. Find the smallest value of n such that $\sum_{k=1}^{n}(3k-5)>100$.

45. How many terms must be added before the series $-1-3-5-7-\cdots$ has a sum less than -75?

46. Piotr devised a week-long study plan to prepare for finals. On the first day, he plans to study for 1 hour, and each successive day he will increase his study time by 30 minutes. How many hours will Piotr have studied after one week?

47. A testing center is designed with 10 seats in the first row, 12 seats in the second row, 14 seats in the third row, and so forth. The testing center has 15 rows of seating. What is the maximum number of students who may be testing at any one time?

48. A brick wall is built with 300 bricks in the first row, 299 bricks in the second row, and each successive row contains one less brick. If the top row contains 177 bricks, what is the total number of bricks required to build the wall?

49. Find the sum $1+2+3+\cdots+1000$.

In Exercises 50 – 55, express the repeating decimal as a fraction of integers.

50. $0.\overline{7}$

51. $0.\overline{13}$

52. $2.\overline{3}$

53. $4.\overline{17}$

54. $10.\overline{159}$

55. $-5.\overline{867}$

In Exercises 56 – 61, compute the future value of the annuity with the given terms. In all cases, assume the payment is made at the end of each month, the interest rate given is the annual rate, and interest is compounded at the end of each month.

56. payments are \$300, interest rate is 2.5%, term is 17 years.

57. payments are \$50, interest rate is 1.0%, term is 30 years.

58. payments are \$100, interest rate is 2.0%, term is 20 years.

59. payments are \$100, interest rate is 2.0%, term is 25 years.

60. payments are \$100, interest rate is 2.0%, term is 30 years.

61. payments are \$100, interest rate is 2.0%, term is 35 years.

62. Discuss with your classmates what goes wrong when trying to find the following sums.[4]

(a) $\sum_{k=1}^{\infty} 2^{k-1}$ (b) $\sum_{k=1}^{\infty} (1.0001)^{k-1}$ (c) $\sum_{k=1}^{\infty} (-1)^{k-1}$

[4] When in doubt, write them out!

7.3 Exercises

1. What is a binomial coefficient and how is it calculated?

2. When is it an advantage to use the Binomial Theorem? Explain.

In Exercises 3 – 8, evaluate the expression.

3. $6!$

4. $\frac{10!}{7!}$

5. $\left(\frac{12}{6}\right)!$

6. $\frac{100!}{99!}$

7. $\frac{7!}{2^3 3!}$

8. $\frac{9!}{4!3!2!}$

In Exercises 9 – 17, evaluate the binomial coefficient.

9. $\binom{6}{2}$

10. $\binom{5}{3}$

11. $\binom{7}{4}$

12. $\binom{8}{3}$

13. $\binom{9}{7}$

14. $\binom{10}{9}$

15. $\binom{117}{0}$

16. $\binom{25}{11}$

17. $\binom{200}{199}$

In Exercises 18 – 29, expand the given binomial.

18. $(4a-b)^3$

19. $(x+2)^5$

20. $(5a+2)^3$

21. $(3a+2b)^3$

22. $(2x+3y)^4$

23. $(2x-1)^4$

24. $(4x+2y)^5$

25. $(3x-2y)^4$

26. $(4x-3y)^5$

27. $\left(\frac{1}{3}x+y^2\right)^3$

28. $\left(\frac{1}{x}+3y\right)^5$

29. $\left(x-x^{-1}\right)^4$

In Exercises 30 – 37, use the Binomial Theorem to find the indicated term.

30. The fourth term of $(2x-3y)^4$

31. The fourth term of $(3x-2y)^5$

32. The third term of $(6x-3y)^7$

33. The eighth term of $(7+5y)^{14}$

34. The seventh term of $(a+b)^{11}$

35. The fifth term of $(x-y)^7$

36. The term containing x^3 in the expansion $(2x-y)^5$

37. The term containing x^{117} in the expansion $(x+2)^{118}$

38. You've just won three tickets to see the new film, '$8.\overline{9}$.' Five of your friends, Brenda, Cindy, Michael, Rachel and Sadie, are interested in seeing it with you. With the help of your classmates, list all the possible ways to distribute your two extra tickets among your five friends. Now suppose you've come down with the flu. List all the different ways you can distribute the three tickets among these five friends. How does this compare with the first list you made? What does this have to do with the fact that $\binom{5}{2}=\binom{5}{3}$?