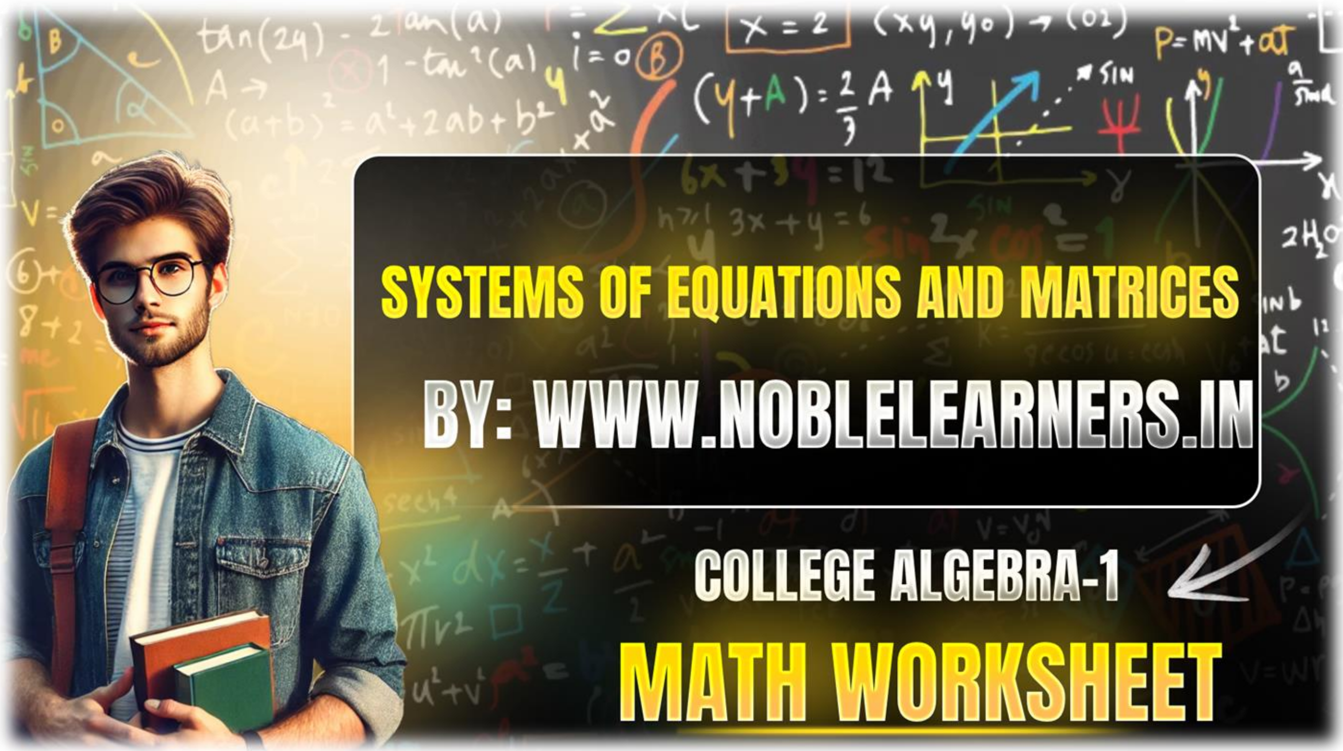

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6.1 Exercises

1. Can a system of linear equations have exactly two solutions? Explain why or why not.

2. Can a system of two non-linear equations have exactly two solutions? What about exactly three? In not, explain why not. If so, sketch the graph of such a system.

In Exercises 3 – 16, solve the given system using substitution and/or elimination. Classify each system as consistent independent, consistent dependent, or inconsistent.

3. $\begin{cases} x+2y=5 \\ x=6 \end{cases}$

4. $\begin{cases} 2y-3x=1 \\ y=-3 \end{cases}$

5. $\begin{cases} x+3y=5 \\ 2x+3y=4 \end{cases}$

6. $\begin{cases} x-2y=3 \\ -3x+6y=-9 \end{cases}$

7. $\begin{cases} 3x-2y=18 \\ 5x+10y=-10 \end{cases}$

8. $\begin{cases} 4x+2y=-10 \\ 3x+9y=0 \end{cases}$

9. $\begin{cases} -2x+5y=-42 \\ 7x+2y=30 \end{cases}$

10. $\begin{cases} 6x-5y=-34 \\ 2x+6y=4 \end{cases}$

11. $\begin{cases} -x+2y=-1 \\ 5x-10y=6 \end{cases}$

12. $\begin{cases} 5x+9y=16 \\ x+2y=4 \end{cases}$

13. $\begin{cases} \dfrac{x+2y}{4}=-5 \\ \dfrac{3x-y}{2}=1 \end{cases}$

14. $\begin{cases} \dfrac{1}{2}x-\dfrac{1}{3}y=-1 \\ 2y-3x=6 \end{cases}$

15. $\begin{cases} x+4y=6 \\ \dfrac{1}{12}x+\dfrac{1}{3}y=\dfrac{1}{2} \end{cases}$

16. $\begin{cases} 3y-\dfrac{3}{2}x=-\dfrac{15}{2} \\ \dfrac{1}{2}x-y=\dfrac{3}{2} \end{cases}$

In Exercises 17 – 20, graph the system of equations and state whether the system has one solution, no solution, or infinite solutions.

17. $\begin{cases} -x+2y=4 \\ 2x-4y=1 \end{cases}$

18. $\begin{cases} x+2y=7 \\ 2x+6y=12 \end{cases}$

19. $\begin{cases} 3x-5y=7 \\ x-2y=3 \end{cases}$

20. $\begin{cases} 3x-2y=5 \\ -9x+6y=-15 \end{cases}$

In Exercises 21 – 32, solve the given system of non-linear equations. Sketch the graph of both equations on the same set of axes to verify the solution set.

21. $\begin{cases} x+y=4 \\ x^2+y^2=9 \end{cases}$

22. $\begin{cases} y=x-3 \\ x^2+y^2=9 \end{cases}$

23. $\begin{cases} y=x \\ x^2+y^2=9 \end{cases}$

24. $\begin{cases} y=-x \\ x^2+y^2=9 \end{cases}$

25. $\begin{cases} x=2 \\ x^2-y^2=9 \end{cases}$

26. $\begin{cases} 4x^2-9y^2=36 \\ 4x^2+9y^2=36 \end{cases}$

27. $\begin{cases} x^2+y^2=25 \\ x^2-y^2=1 \end{cases}$

28. $\begin{cases} x^2-y=4 \\ x^2+y^2=4 \end{cases}$

29. $\begin{cases} x^2+y^2=4 \\ x^2-y=5 \end{cases}$

30. $\begin{cases} x^2+y^2=16 \\ 16x^2+4y^2=64 \end{cases}$

31. $\begin{cases} x^2+y^2=16 \\ 9x^2-16y^2=144 \end{cases}$

32. $\begin{cases} x^2+y^2=16 \\ \frac{1}{9}y^2-\frac{1}{16}x^2=1 \end{cases}$

In Exercises 33 – 40, solve the given system of non-linear equations.

33. $\begin{cases} x^2+y^2=16 \\ x-y=2 \end{cases}$

34. $\begin{cases} x^2-y^2=1 \\ x^2+4y^2=4 \end{cases}$

35. $\begin{cases} x+2y^2=2 \\ x^2+4y^2=4 \end{cases}$

36. $\begin{cases} (x-2)^2+y^2=1 \\ x^2+4y^2=4 \end{cases}$

37. $\begin{cases} x^2+y^2=25 \\ y-x=1 \end{cases}$

38. $\begin{cases} x^2+y^2=25 \\ x^2+(y-3)^2=10 \end{cases}$

39. $\begin{cases} y=x^3+8 \\ y=10x-x^2 \end{cases}$

40. $\begin{cases} x^3-10x+y=5 \\ x-y=-5 \end{cases}$

6.2 Exercises

1. Can a linear system of three equations have exactly two solutions? Explain why or why not.
2. What is the geometric interpretation of a system of linear equations in three variables that is independent? How many solutions are there?

In Exercises 3 – 20, solve the system, if possible. State any solutions and classify each system as consistent independent, consistent dependent, or inconsistent.

3. $\begin{cases} x+y+z=3 \\ 2x-y+z=0 \\ -3x+5y+7z=7 \end{cases}$

4. $\begin{cases} 4x-y+z=5 \\ 2y+6z=30 \\ x+z=5 \end{cases}$

5. $\begin{cases} 4x-y+z=5 \\ 2y+6z=30 \\ x+z=6 \end{cases}$

6. $\begin{cases} x+y+z=-17 \\ y-3z=0 \end{cases}$

7. $\begin{cases} x+2y+z=7 \\ -y+3z=9 \end{cases}$

8. $\begin{cases} x-2y+3z=7 \\ -3x+y+2z=-5 \\ 2x+2y+z=3 \end{cases}$

9. $\begin{cases} 3x-2y+z=-5 \\ x+3y-z=12 \\ x+y+2z=0 \end{cases}$

10. $\begin{cases} 2x-y+z=-1 \\ 4x+3y+5z=1 \\ 5y+3z=4 \end{cases}$

11. $\begin{cases} x-y+z=-4 \\ -3x+2y+4z=-5 \\ x-5y+2z=-18 \end{cases}$

12. $\begin{cases} 2x-4y+z=-7 \\ x-2y+2z=-2 \\ -x+4y-2z=3 \end{cases}$

13. $\begin{cases} 2x-y+z=1 \\ 2x+2y-z=1 \\ 3x+6y+4z=9 \end{cases}$

14. $\begin{cases} x-3y-4z=3 \\ 3x+4y-z=13 \\ 2x-19y-19z=2 \end{cases}$

15. $\begin{cases} x+y+z=4 \\ 2x-4y-z=-1 \\ x-y=2 \end{cases}$

16. $\begin{cases} x-y+z=8 \\ 3x+3y-9z=-6 \\ 7x-2y+5z=39 \end{cases}$

17. $\begin{cases} 2x-3y+z=-1 \\ 4x-4y+4z=-13 \\ 6x-5y+7z=-25 \end{cases}$

18. $\begin{cases} 2x_1+x_2-12x_3-x_4=16 \\ -x_1+x_2+12x_3-4x_4=-5 \\ 3x_1+2x_2-16x_3-3x_4=25 \\ x_1+2x_2-5x_4=11 \end{cases}$

19. $\begin{cases} x_1-x_3=-2 \\ 2x_2-x_4=0 \\ x_1-2x_2+x_3=0 \\ -x_3+x_4=1 \end{cases}$

20. $\begin{cases} x_1-x_2-5x_3+3x_4=-1 \\ x_1+x_2+5x_3-3x_4=0 \\ x_2+5x_3-3x_4=1 \\ x_1-2x_2-10x_3+6x_4=-1 \end{cases}$

21. You inherit one million dollars. You invest it all in three accounts for one year. The first account pays 3% compounded annually, the second account pays 4% compounded annually, and the third account pays 2% compounded annually. After one year, you earn $34,000 in interest. If you invest four times the money into the account that pays 3% compared to 2%, how much did you invest in each account?

22. You inherit one hundred thousand dollars. You invest it all in three accounts for one year. The first account pays 4% compounded annually, the second account pays 3% compounded annually, and the third account pays 2% compounded annually. After one year, you earn $3,650 in interest. If you invest five times the money into the account that pays 4% compared to 3%, how much did you invest in each account?

23. Find the quadratic function passing through the points $(-1,-4)$, $(1,6)$, and $(3,0)$.

24. Find the quadratic function passing through the points $(1,-1)$, $(3,-1)$, and $(-2,14)$.

25. Find the equation in standard form of the circle passing through the points $(-2,5)$, $(5,4)$, and $(1,-4)$.

26. The top three countries in oil consumption in a certain year are as follows: United States, Japan, and China. In millions of barrels per day, the top three countries consumed 39.8% of the world's consumed oil. The United States consumed 0.7% more than four times China's consumption. The United States consumed 5% more than triple Japan's consumption. What percent of the world oil consumption did the United States, Japan, and China consume?

27. The top three countries in oil production in the same year are Saudi Arabia, United States, and Russia. In millions of barrels per day, the top three countries produced 31.4% of the world's produced oil. Saudi Arabia and the United States combined for 22.1% of the world's production, and Saudi Arabia produced 2% more oil than Russia. What percent of the world oil production did Saudi Arabia, the United States, and Russia produce?

28. The top three sources of oil imports for the United States in the same year were Saudi Arabia, Mexico, and Canada. These top three countries accounted for 47% of oil imports. The United States imported 1.8% more from Saudi Arabia than they did from Mexico, and 1.7% more from Saudi Arabia than they did from Canada. What percentage of United States oil imports were from these three countries?

29. The top three oil producers in the United States in a certain year are the Gulf of Mexico, Texas, and Alaska. The three regions were responsible for 64% of the United States oil production. The Gulf of Mexico and Texas combined for 47% of oil production. Texas produced 3% more than Alaska. What percent of United States oil production came from these regions?

30. At a family reunion, there were only blood relatives, consisting of children, parents, and grandparents, in attendance. There were 400 people total. There were twice as many parents as grandparents, and 50 more children than parents. How many children, parents, and grandparents were in attendance?

31. An animal shelter has a total of 350 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 5 less than one-half the number of cats, and there are 20 more cats than dogs, how many of each type of animal are at the shelter?

32. Your roommate, Prasantha, offered to buy groceries for you and your other roommate. The total bill was $82. She forgot to save the individual receipts but remembered that your groceries were $0.05 cheaper than half of her groceries, and that your other roommate's groceries were $2.10 more than your groceries. How much was each of your shares of the groceries?

33. Three co-workers have jobs as warehouse manager, office manager, and truck driver. The sum of the annual salaries of the warehouse manager and office manager is $82,000. The office manager makes $4,000 more that the truck driver annually. The annual salaries of the warehouse manager and the truck driver total $78,000. What is the annual salary of each of the co-workers?

34. At a carnival, $2,941.25 in receipts was taken by the end of the day. The cost of a child's ticket was $20.50, an adult ticket was $29.75, and a senior citizen ticket was $15.25. There were twice as many senior citizens as adults in attendance, and 20 more children than senior citizens. How many children, adult, and senior citizen tickets were sold?

35. A local band sells out for their concert. They sell all 1,175 tickets for a total purse of $28,112.50. The tickets were priced at $20 for students, $22.50 for children, and $29 for adults. If the band sold twice as many adult as child tickets, how many of each type were sold?

36. In a bag, a child has 325 coins worth $19.50. There are three types of coins: pennies, nickels, and dimes. If the bag contains the same number of nickels as dimes, how many of each type of coin is in the bag?

6.3 Exercises

1. What are two different row operations that can be used to obtain a leading 1 in the first row of the matrix $\left[\begin{array}{cc|c} 9 & 3 & 0 \\ 1 & -2 & 6 \end{array}\right]$?

2. What does a row of 0's in an augmented matrix tell you about the system of equations it is representing?

In Exercises 3 – 6, state whether the given matrix is in reduced row echelon form, row echelon form only, or neither of these.

3. $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 3 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 3 & -1 & 1 & 3 \\ 2 & -4 & 3 & 16 \\ 1 & -1 & 1 & 5 \end{array}\right]$

5. $\left[\begin{array}{ccc|c} 1 & 1 & 4 & 3 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 \end{array}\right]$

6. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

In Exercises 7 – 10, the matrices are in reduced row echelon form. Determine the solution of the corresponding system of linear equations or state that the system is inconsistent.

7. $\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 7 \end{array}\right]$

8. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & 19 \end{array}\right]$

9. $\left[\begin{array}{ccc|c} 1 & 0 & 9 & -3 \\ 0 & 1 & -4 & 20 \\ 0 & 0 & 0 & 0 \end{array}\right]$

10. $\left[\begin{array}{ccc|c} 1 & 0 & 9 & -3 \\ 0 & 1 & -4 & 20 \\ 0 & 0 & 0 & 1 \end{array}\right]$

In Exercises 11 – 26, solve the systems of linear equations using elementary row operations on an augmented matrix.

11. $\begin{cases} -5x + y = 17 \\ x + y = 5 \end{cases}$

12. $\begin{cases} 2x - 3y = -9 \\ 5x + 4y = 58 \end{cases}$

13. $\begin{cases} 2x + 3y = 12 \\ 4x + y = 14 \end{cases}$

14. $\begin{cases} 3x + 4y = 12 \\ -6x - 8y = -24 \end{cases}$

15. $\begin{cases} x+y+z=3 \\ 2x-y+z=0 \\ -3x+5y+7z=7 \end{cases}$

16. $\begin{cases} 4x-y+z=5 \\ 2x+6z=30 \\ x+z=5 \end{cases}$

17. $\begin{cases} x-2y+3z=7 \\ -3x+y+2z=-5 \\ 2x+2y+z=3 \end{cases}$

18. $\begin{cases} 3x-2y+z=-5 \\ x+3y-z=12 \\ x+y+2z=0 \end{cases}$

19. $\begin{cases} 2x-y+z=-1 \\ 4x+3y+5z=1 \\ 5y+3z=4 \end{cases}$

20. $\begin{cases} x-y+z=-4 \\ -3x+2y+4z=-5 \\ x-5y+2z=-18 \end{cases}$

21. $\begin{cases} 2x-4y+z=-7 \\ x-2y+2z=-2 \\ -x+4y-2z=3 \end{cases}$

22. $\begin{cases} 2x-y+z=1 \\ 2x+2y-z=1 \\ 3x+6y+4z=9 \end{cases}$

23. $\begin{cases} x-3y-4z=3 \\ 3x+4y-z=13 \\ 2x-19y-19z=2 \end{cases}$

24. $\begin{cases} x+y+z=4 \\ 2x-4y-z=-1 \\ x-y=2 \end{cases}$

25. $\begin{cases} x-y+z=8 \\ 2x+3y-9z=-6 \\ 7x-2y+5z=39 \end{cases}$

26. $\begin{cases} 2x-3y+z=-1 \\ 4x-4y+4z=-13 \\ 6x-5y+7z=-25 \end{cases}$

27. At the local buffet, 22 diners (5 of whom were children) feasted for \$162.25, before taxes. If the kids buffet is \$4.50, the basic buffet is \$7.50, and the deluxe buffet (with crab legs) is \$9.25, how many diners chose the deluxe buffet?

28. Jianming wants to make a party mix consisting of almonds (which cost \$7 per pound), cashews (which cost \$5 per pound), and peanuts (which cost \$2 per pound). If he wants to make a 10 pound mix with a budget of \$35, what are the possible combinations of almonds, cashews, and peanuts?

29. Find the quadratic function passing through the points $(-2,1)$, $(1,4)$, and $(3,-2)$.

30. Find two different row echelon forms for the matrix $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 12 & 8 \end{array}\right]$.

6.4 Exercises

1. Can we add any two matrices together? If so, explain why; if not, explain why not and give an example of two matrices that cannot be added together.

2. Can any two matrices of the same size be multiplied? If so, explain why; if not, explain why not and give an example of two matrices of the same size that cannot be multiplied together.

For each pair of matrices A and B in Exercises 3 – 9, find the following, if defined.

(a) $3A$ (b) $-B$ (c) A^2

(d) $A-2B$ (e) AB (f) BA

3. $A=\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 5 & -2 \\ 4 & 8 \end{bmatrix}$

4. $A=\begin{bmatrix} -1 & 5 \\ -3 & 6 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 10 \\ -7 & 1 \end{bmatrix}$

5. $A=\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 7 & 0 & 8 \\ -3 & 1 & 4 \end{bmatrix}$

6. $A=\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$, $B=\begin{bmatrix} -1 & 3 & -5 \\ 7 & -9 & 11 \end{bmatrix}$

7. $A=\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

8. $A=\begin{bmatrix} 1 & -2 \\ -3 & 4 \\ 5 & -6 \end{bmatrix}$, $B=\begin{bmatrix} -5 & 1 & 8 \end{bmatrix}$

9. $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -2 \\ -7 & 1 & -1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 & 1 \\ 17 & 33 & 19 \\ 10 & 19 & 11 \end{bmatrix}$

In Exercises 10 – 23, use the following matrices to compute the indicated operation or state that the indicated operation is undefined.

$$A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B=\begin{bmatrix} 0 & -3 \\ -5 & 2 \end{bmatrix} \quad C=\begin{bmatrix} 10 & -\frac{11}{2} & 0 \\ \frac{3}{5} & 5 & 9 \end{bmatrix} \quad D=\begin{bmatrix} 7 & -13 \\ -\frac{4}{3} & 0 \\ 6 & 8 \end{bmatrix} \quad E=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -9 \\ 0 & 0 & -5 \end{bmatrix}$$

10. $7B-4A$
11. AB
12. BA
13. $E+D$
14. ED
15. $CD+2I_2A$
16. $A-4I_2$
17. A^2-B^2
18. $(A+B)(A-B)$
19. $A^2-5A-2I_2$
20. $E^2+5E-36I_3$
21. EDC

22. CDE 23. $ABCEDI_2$

24. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$, and $E_3 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$. Compute E_1A, E_2A, and E_3A. What effect did each of the E_i matrices have on the rows of A? Create E_4 so that its effect on A is to multiply the bottom row by −6. How would you extend this idea to matrices with more than two rows?

25. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -3 \\ -5 & 2 \end{bmatrix}$. Compare $(A+B)^2$ to $A^2 + 2AB + B^2$. Discuss with your classmates what constraints must be placed on two arbitrary matrices A and B so that both $(A+B)^2$ and $A^2 + 2AB + B^2$ exist. When will $(A+B)^2 = A^2 + 2AB + B^2$? In general, what is the correct formula for $(A+B)^2$?

In Exercises 26 – 30, consider the following definitions. A square matrix is said to be an **upper triangular matrix** if all of its entries below the main diagonal are zero and it is said to be a **lower triangular matrix** if all of its entries above the main diagonal are zero. For example,

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -9 \\ 0 & 0 & -5 \end{bmatrix}$$

is an upper triangular matrix, whereas

$$F = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$$

is a lower triangular matrix (zeros are allowed on the main diagonal). Discuss the following questions with your classmates.

26. Give an example of a matrix that is neither upper triangular nor lower triangular.

27. Is the product of two $n \times n$ upper triangular marices always upper triangular?

28. Is the product of two $n \times n$ lower triangular matrices always lower triangular?

29. Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, write A as LU where L is a lower triangular matrix and U is an upper triangular matrix.

30. Are there any matrices that are simultaneously upper and lower triangular?

6.5 Exercises

1. In a previous section, we showed examples in which matrix multiplication is not commutative, that is, $AB \neq BA$. Explain why matrix multiplication is commutative for matrix inverses, that is, $A^{-1}A = AA^{-1}$.

2. Does every 2×2 matrix have an inverse? Explain why or why not. Explain what condition is necessary for an inverse to exist.

In Exercises 3 – 8, verify that the matrix A is the inverse of the matrix B.

3. $A=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

4. $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B=\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

5. $A=\begin{bmatrix} 4 & 5 \\ 7 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 0 & \frac{1}{7} \\ \frac{1}{5} & -\frac{4}{35} \end{bmatrix}$

6. $A=\begin{bmatrix} -2 & \frac{1}{2} \\ 3 & -1 \end{bmatrix}$, $B=\begin{bmatrix} -2 & -1 \\ -6 & -4 \end{bmatrix}$

7. $A=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$, $B=\frac{1}{2}\begin{bmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

8. $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 1 & 6 & 9 \end{bmatrix}$, $B=\frac{1}{4}\begin{bmatrix} 6 & 0 & -2 \\ 17 & -3 & -5 \\ -12 & 2 & 4 \end{bmatrix}$

In Exercises 9 – 22, find the inverse of the matrix or state that the matrix is singular (not invertible).

9. $A=\begin{bmatrix} 3 & -2 \\ 1 & 9 \end{bmatrix}$

10. $B=\begin{bmatrix} -2 & 2 \\ 3 & 1 \end{bmatrix}$

11. $C=\begin{bmatrix} -3 & 7 \\ 9 & 2 \end{bmatrix}$

12. $D=\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

13. $E=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

14. $F=\begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$

15. $G=\begin{bmatrix} 6 & 15 \\ 14 & 35 \end{bmatrix}$

16. $H=\begin{bmatrix} 2 & -1 \\ 16 & -9 \end{bmatrix}$

17. $J=\begin{bmatrix} 1 & 0 & 6 \\ -2 & 1 & 7 \\ 3 & 0 & 2 \end{bmatrix}$

18. $K=\begin{bmatrix} 0 & 1 & -3 \\ 4 & 1 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

19. $L = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 1 \\ -2 & -4 & -5 \end{bmatrix}$

20. $M = \begin{bmatrix} 3 & 0 & 4 \\ 2 & -1 & 3 \\ -3 & 2 & -5 \end{bmatrix}$

21. $N = \begin{bmatrix} 4 & 6 & -3 \\ 3 & 4 & -3 \\ 1 & 2 & 6 \end{bmatrix}$

22. $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 11 \\ 3 & 4 & 19 \end{bmatrix}$

In Exercises 23 – 30, use a matrix inverse to solve the system of linear equations.

23. $\begin{cases} 3x + 7y = 26 \\ 5x + 12y = 39 \end{cases}$

24. $\begin{cases} 3x + 7y = 0 \\ 5x + 12y = -1 \end{cases}$

25. $\begin{cases} 3x + 7y = -7 \\ 5x + 12y = 5 \end{cases}$

26. $\begin{cases} 5x - 6y = -61 \\ 4x + 3y = -2 \end{cases}$

27. $\begin{cases} 8x + 4y = -100 \\ 3x - 4y = 1 \end{cases}$

28. $\begin{cases} 3x - 2y = 6 \\ -x + 5y = -2 \end{cases}$

29. $\begin{cases} -3x - 4y = 9 \\ 12x + 4y = -6 \end{cases}$

30. $\begin{cases} -2x + 3y = \dfrac{3}{10} \\ -x + 5y = \dfrac{1}{2} \end{cases}$

In Exercises 31 – 33, use the inverse of M from **Exercise 20** to solve the system of linear equations.

31. $\begin{cases} 3x + 4z = 1 \\ 2x - y + 3z = 0 \\ -3x + 2y - 5z = 0 \end{cases}$

32. $\begin{cases} 3x + 4z = 0 \\ 2x - y + 3z = 1 \\ -3x + 2y - 5z = 0 \end{cases}$

33. $\begin{cases} 3x + 4z = 0 \\ 2x - y + 3z = 0 \\ -3x + 2y - 5z = 1 \end{cases}$

34. Solve the following system of linear equations using an inverse matrix.

$$\begin{cases} x_1 + 2x_2 + 3x_4 = 2 \\ x_2 + x_3 + x_4 = -1 \\ x_2 + x_4 = 5 \\ x_1 + 2x_2 + 2x_4 = 0 \end{cases}$$

The inverse matrix is provided for you as follows: $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & -2 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$.

35. Solve the following system of linear equations using an inverse matrix.

$$\begin{cases} x+y-3w=7 \\ x-2z=1 \\ 2y-z+w=4 \\ 2x+3y-2w=-3 \end{cases}$$

The inverse matrix is provided for you as follows: $\begin{bmatrix} 1 & 1 & 0 & -3 \\ 1 & 0 & -2 & 0 \\ 0 & 2 & -1 & 1 \\ 2 & 3 & 0 & -2 \end{bmatrix}^{-1} = \frac{1}{21}\begin{bmatrix} -14 & 7 & -14 & 14 \\ 2 & -4 & 8 & 1 \\ -7 & -7 & -7 & 7 \\ -11 & 1 & -2 & 5 \end{bmatrix}$.

36. Solve the following system of linear equations using an inverse matrix.

$$\begin{cases} x+y+z+w=5 \\ x-y+z-w=2 \\ x+2y+3z+4w=0 \\ x-2y+3z-4w=9 \end{cases}$$

The inverse matrix is provided for you as follows: $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 1 & -2 & 3 & -4 \end{bmatrix}^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 3 & -1 & -1 \\ 4 & -4 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -2 & 2 & 1 & -1 \end{bmatrix}$.

37. Matrices can be used in cryptography. Suppose we wish to encode the message 'BIGFOOT LIVES'. We start by assigning a number to each letter of the alphabet, say $A=1$, $B=2$, and so on. We reserve 0 to act as a space. Hence, our message 'BIGFOOT LIVES' corresponds to the string of numbers '2, 9, 7, 6, 15, 15, 20, 0, 12, 9, 22, 5, 19'. To encode this message, we use an invertible matrix. Any invertible matrix will do, but for this exercise, we choose

$$A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -2 \\ -7 & 1 & -1 \end{bmatrix}$$

Since A is a 3×3 matrix, we encode our message string into a matrix M with 3 rows. To do this, we take the first three numbers, 2, 9, 7, and make them our first column, the next three numbers, 6, 15, 15, and make them our second column, and so on. We put 0's to round out the matrix.

$$M=\begin{bmatrix} 2 & 6 & 20 & 9 & 19 \\ 9 & 15 & 0 & 22 & 0 \\ 7 & 15 & 12 & 5 & 0 \end{bmatrix}$$

To encode the message, we find the product AM.

$$AM = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 1 & -2 \\ -7 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 20 & 9 & 19 \\ 9 & 15 & 0 & 22 & 0 \\ 7 & 15 & 12 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 42 & 100 & -23 & 38 \\ 1 & 3 & 36 & 39 & 57 \\ -12 & -42 & -152 & -46 & -133 \end{bmatrix}$$

So our coded message is '12, 1, −12, 42, 3, −42, 100, 36, −152, −23, 39, −46, 38, 57, −133'. To decode this message, we start with this string of numbers, construct a message matrix as we did earlier (we should get the matrix AM again) and then multiply by A^{-1}.

(a) Find A^{-1}.

(b) Use A^{-1} to decode the message and check that this method actually works.

(c) Decode the message '14, 37, −76, 128, 21, −151, 31, 65, −140'.

(d) Choose another invertible matrix and encode and decode your own message.

6.6 Exercises

1. Can we always evaluate the determinant of a square matrix? Explain why or why not.

2. Examining Cramer's Rule, explain why there is no unique solution to the system when the determinant of the coefficient matrix is 0. For simplicity, use a 2×2 matrix.

In Exercises 3 – 22, compute the determinant of the given matrix. (Most of these matrices appeared in the **6.5 Exercises**.)

3. $A = \begin{bmatrix} 3 & -2 \\ 1 & 9 \end{bmatrix}$

4. $B = \begin{bmatrix} -2 & 2 \\ 3 & 1 \end{bmatrix}$

5. $C = \begin{bmatrix} -3 & 7 \\ 9 & 2 \end{bmatrix}$

6. $D = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

7. $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

8. $F = \begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$

9. $G = \begin{bmatrix} 6 & 15 \\ 14 & 35 \end{bmatrix}$

10. $H = \begin{bmatrix} 2 & -1 \\ 16 & -9 \end{bmatrix}$

11. $J = \begin{bmatrix} 1 & 0 & 6 \\ -2 & 1 & 7 \\ 3 & 0 & 2 \end{bmatrix}$

12. $K = \begin{bmatrix} 0 & 1 & -3 \\ 4 & 1 & 0 \\ 1 & 0 & 5 \end{bmatrix}$

13. $L = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 1 \\ -2 & -4 & -5 \end{bmatrix}$

14. $M = \begin{bmatrix} 3 & 0 & 4 \\ 2 & -1 & 3 \\ -3 & 2 & -5 \end{bmatrix}$

15. $N = \begin{bmatrix} 4 & 6 & -3 \\ 3 & 4 & -3 \\ 1 & 2 & 6 \end{bmatrix}$

16. $P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 11 \\ 3 & 4 & 19 \end{bmatrix}$

17. $Q = \begin{bmatrix} x & x^2 \\ 1 & 2x \end{bmatrix}$

18. $R = \begin{bmatrix} i & j & k \\ -1 & 0 & 5 \\ 9 & -4 & -2 \end{bmatrix}$

19. $S = \begin{bmatrix} 1 & 0 & -3 & 0 \\ 2 & -2 & 8 & 7 \\ -5 & 0 & 16 & 0 \\ 1 & 0 & 4 & 1 \end{bmatrix}$

20. $T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

21. $V = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 2 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

22. $W = \begin{bmatrix} 4 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 10 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$

In Exercises 23 – 34, use Cramer's Rule to solve the system of linear equations. (Some of these systems appeared in the **6.5 Exercises**.)

23. $\begin{cases} 3x+7y=26 \\ 5x+12y=39 \end{cases}$

24. $\begin{cases} 3x+7y=0 \\ 5x+12y=-1 \end{cases}$

25. $\begin{cases} 3x+7y=-7 \\ 5x+12y=5 \end{cases}$

26. $\begin{cases} 5x-6y=-61 \\ 4x+3y=-2 \end{cases}$

27. $\begin{cases} 8x+4y=-100 \\ 3x-4y=1 \end{cases}$

28. $\begin{cases} 3x-2y=6 \\ -x+5y=-2 \end{cases}$

29. $\begin{cases} -3x-4y=9 \\ 12x+4y=-6 \end{cases}$

30. $\begin{cases} -2x+3y=\frac{3}{10} \\ -x+5y=\frac{1}{2} \end{cases}$

31. $\begin{cases} x+y+z=3 \\ 2x-y+z=0 \\ -3x+5y+7z=7 \end{cases}$

32. $\begin{cases} 3x+y-2z=10 \\ 4x-y+z=5 \\ x-3y-4z=-1 \end{cases}$

33. $\begin{cases} x+2y-4z=-1 \\ 7x+3y+5z=26 \\ -2x-6y+7z=-6 \end{cases}$

34. $\begin{cases} -5x+2y-4z=-47 \\ 4x-3y-z=-94 \\ 3x-3y+2z=94 \end{cases}$

In Exercises 35 – 40, use Cramer's Rule to solve the system of linear equations for the indicated variable.

35. Solve for x: $\begin{cases} 4x+5y-z=-7 \\ -2x-9y+2z=8 \\ 5y+7z=21 \end{cases}$

36. Solve for y: $\begin{cases} 4x-3y+4z=10 \\ 5x-2z=-2 \\ 3x+2y-5z=-9 \end{cases}$

37. Solve for z: $\begin{cases} 4x-2y+3z=6 \\ -6x+y=-2 \\ 2x+7y+8z=24 \end{cases}$

38. Solve for x: $\begin{cases} -4x-3y-8z=-7 \\ 2x-9y+5z=\frac{1}{2} \\ 5x-6y-5z=-2 \end{cases}$

39. Solve for x_4: $\begin{cases} x_1-x_3=-2 \\ 2x_2-x_4=0 \\ x_1-2x_2+x_3=0 \\ -x_3+x_4=1 \end{cases}$

40. Solve for x_4: $\begin{cases} 4x_1+x_2=4 \\ x_2-3x_3=1 \\ 10x_1+x_3+x_4=0 \\ -x_2+x_3=-3 \end{cases}$

41. Carl's Sasquatch Attack! game card collection is a mixture of common and rare cards. Each common card is worth \$0.25 while each rare card is worth \$0.75. If his entire 117 card collection is worth \$48.75, how many of each kind of card does he own?

42. Brenda's Exotic Animal Rescue houses snakes, tarantulas and scorpions. When asked how many animals of each kind she boards, Brenda answered: 'We board 49 total animals, and I am responsible for each of their 272 legs and 28 tails.' How many of each animal does the Rescue board? (Recall: Tarantulas have 8 legs and no tails; scorpions have 8 legs and one tail; snakes have no legs and one tail.)

6.7 Exercises

1. Can any quotient of polynomials be decomposed into at least two partial fractions? If so, explain why, and if not, give an example.
2. How can you check that you decomposed a partial fraction correctly?

In Exercises 3 – 16, find the partial fraction decomposition for the rational expressions with denominators that contain non-repeated linear factors.

3. $\dfrac{5x+16}{x^2+10x+24}$

4. $\dfrac{3x-79}{x^2-5x-24}$

5. $\dfrac{-x-24}{x^2-2x-24}$

6. $\dfrac{10x+47}{x^2+7x+10}$

7. $\dfrac{x}{6x^2+25x+25}$

8. $\dfrac{32x-11}{20x^2-13x+2}$

9. $\dfrac{x+1}{x^2+7x+10}$

10. $\dfrac{5x}{x^2-9}$

11. $\dfrac{10x}{x^2-25}$

12. $\dfrac{6x}{x^2-4}$

13. $\dfrac{2x-3}{x^2-6x+5}$

14. $\dfrac{4x-1}{x^2-x-6}$

15. $\dfrac{4x+3}{x^2+8x+15}$

16. $\dfrac{3x-1}{x^2-5x+6}$

In Exercises 17 – 27, find the partial fraction decomposition for the rational expressions with denominators that contain repeated linear factors.

17. $\dfrac{-5x-19}{(x+4)^2}$

18. $\dfrac{x}{(x-2)^2}$

19. $\dfrac{7x+14}{(x+3)^2}$

20. $\dfrac{-24x-27}{(4x+5)^2}$

21. $\dfrac{-24x-27}{(6x-7)^2}$

22. $\dfrac{5-x}{(x-7)^2}$

23. $\dfrac{5x+14}{2x^2+12x+18}$

24. $\dfrac{5x^2+20x+8}{2x(x+1)^2}$

25. $\dfrac{4x^2+55x+25}{5x(3x+5)^2}$

26. $\dfrac{54x^3+127x^2+80x+16}{2x^2(3x+2)^2}$

27. $\dfrac{x^3-5x^2+12x+144}{x^2(x^2+12x+36)}$

In Exercises 28 – 40, find the partial fraction decomposition for the rational expressions with denominators that contain non-repeated irreducible quadratic factors.

28. $\dfrac{4x^2+6x+11}{(x+2)(x^2+x+3)}$

29. $\dfrac{4x^2+9x+23}{(x-1)(x^2+6x+11)}$

30. $\dfrac{-2x^2+10x+4}{(x-1)(x^2+3x+8)}$

31. $\dfrac{x^2+3x+1}{(x+1)(x^2+5x-2)}$

32. $\dfrac{4x^2+17x-1}{(x+3)(x^2+6x+1)}$

33. $\dfrac{4x^2}{(x+5)(x^2+7x-5)}$

34. $\dfrac{4x^2+5x+3}{x^3-1}$

35. $\dfrac{-5x^2+18x-4}{x^3+8}$

36. $\dfrac{3x^2-7x+33}{x^3+27}$

37. $\dfrac{x^2+2x+40}{x^3-125}$

38. $\dfrac{4x^2+4x+12}{8x^3-27}$

39. $\dfrac{-50x^2+5x-3}{125x^3-1}$

40. $\dfrac{-2x^3-30x^2+36x+216}{x^4+216x}$

In Exercises 41 – 51, find the partial fraction decomposition for the rational expressions with denominators that contain repeated irreducible quadratic factors.

41. $\dfrac{2x-9}{(x^2-x)^2}$

42. $\dfrac{3x^3+2x^2+14x+15}{(x^2+4)^2}$

43. $\dfrac{x^3+6x^2+5x+9}{(x^2+1)^2}$

44. $\dfrac{x^3-x^2+x-1}{(x^2-3)^2}$

45. $\dfrac{x^2+5x+5}{(x^2+2)^2}$

46. $\dfrac{x^3+2x^2+4x}{(x^2+2x+9)^2}$

47. $\dfrac{x^2+25}{(x^2+3x+25)^2}$

48. $\dfrac{2x^3+11x^2+7x+70}{(2x^2+x+14)^2}$

49. $\dfrac{5x+2}{x(x^2+4)^2}$

50. $\dfrac{x^4+x^3+8x^2+6x+36}{x(x^2+6)^2}$

51. $\dfrac{5x^3-2x+1}{(x^2+2)^2}$

In Exercises 52 – 55, use division to decompose the improper rational expression into the sum of a polynomial and a proper rational expression.

52. $\dfrac{x^2+x-1}{x-1}$

53. $\dfrac{6x^2+5x-14}{3x+2}$

54. $\dfrac{x^3+8}{x^2+4}$

55. $\dfrac{x^3-x^2-7x+10}{x^2+2x+7}$