# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can identify quadratic functions, evaluate them accurately, and analyze their algebraic structure.

*Equivalent exact forms are accepted unless a specific form is requested.*

**1.** The highest exponent of $x$ is $2$, so $f(x) = 3x^2 - 2x + 7$ is a $\boxed{\text{quadratic function}}$.

**2.** Substitute $x = -3$:

$$q(-3) = 2(-3)^2 + 5(-3) - 1 = 18 - 15 - 1 = \boxed{2}.$$

**3.** Write the terms in descending powers of $x$:

$$4 - 3x^2 + 8x = \boxed{-3x^2 + 8x + 4}.$$

Thus $\boxed{a = -3}$, $\boxed{b = 8}$, and $\boxed{c = 4}$.

**4.** For $q(x) = x^2 - 4x + 6$:

$$q(0) = 6, \qquad q(1) = 1 - 4 + 6 = 3, \qquad q(2) = 4 - 8 + 6 = 2.$$

Therefore $\boxed{q(0) = 6,\ q(1) = 3,\ q(2) = 2}$.

**5.** Replace every $x$ by $-x$:

$$q(-x) = (-x)^2 + 3(-x) - 5 = \boxed{x^2 - 3x - 5}.$$

**6.** First compute $q(x+1)$:

$$q(x+1) = (x+1)^2 - 2(x+1) + 4 = x^2 + 3.$$

Hence

$$q(x+1) - q(x) = x^2 + 3 - (x^2 - 2x + 4) = \boxed{2x - 1}.$$

## Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can expand, factor, and solve quadratic equations using the Zero Product Property.

**7.** Distribute each term:

$$(x-5)(x+2) = x^2 + 2x - 5x - 10 = \boxed{x^2 - 3x - 10}.$$

**8.** Two integers with product $20$ and sum $9$ are $4$ and $5$:

$$x^2 + 9x + 20 = \boxed{(x+4)(x+5)}.$$

**9.** Two integers with product $-16$ and sum $-6$ are $-8$ and $2$:

$$x^2 - 6x - 16 = \boxed{(x-8)(x+2)}.$$

**10.** Split the middle term and group:

$$3x^2 + x - 2 = 3x^2 + 3x - 2x - 2 = \boxed{(3x-2)(x+1)}.$$

**11.** Factor and set each factor equal to zero:

$$x^2 - 7x + 12 = (x-3)(x-4) = 0.$$

Therefore $\boxed{x = 3 \text{ or } x = 4}$.

**12.** Factor:

$$2x^2 - 5x - 3 = (2x+1)(x-3) = 0.$$

Thus $\boxed{x = -\frac{1}{2} \text{ or } x = 3}$.

## Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can solve quadratic equations by square roots and by completing the square.

**13.** Divide by $5$ and take both square roots:

$$5x^2 = 80 \Rightarrow x^2 = 16 \Rightarrow \boxed{x = \pm 4}.$$

**14.** Take both square roots:

$$(x-6)^2 = 49 \Rightarrow x - 6 = \pm 7.$$

Therefore $\boxed{x = 13 \text{ or } x = -1}$.

**15.** Half of $10$ is $5$, and $5^2 = 25$:

$$x^2 + 10x + 7 = (x^2 + 10x + 25) - 18 = \boxed{(x+5)^2 - 18}.$$

**16.** Half of $-8$ is $-4$, and $(-4)^2 = 16$:

$$x^2 - 8x + 3 = (x^2 - 8x + 16) - 13 = \boxed{(x-4)^2 - 13}.$$

**17.** Complete the square:

$$\begin{aligned} x^2 + 6x - 7 &= 0 \\ x^2 + 6x + 9 &= 16 \\ (x+3)^2 &= 16 \\ x + 3 &= \pm 4. \end{aligned}$$

Thus $\boxed{x = 1 \text{ or } x = -7}$.

**18.** Complete the square:

$$x^2 - 4x - 1 = 0 \Rightarrow (x-2)^2 = 5.$$

Therefore $\boxed{x = 2 \pm \sqrt{5}}$.

# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can apply the quadratic formula and use the discriminant to classify solutions.

**19.** For $3x^2 + 2x - 8 = 0$, $a = 3$, $b = 2$, and $c = -8$:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-8)}}{2(3)} = \frac{-2 \pm 10}{6}.$$

Hence $\boxed{x = \frac{4}{3} \text{ or } x = -2}$.

**20.** For $2x^2 + 5x + 1 = 0$:

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{4} = \boxed{\frac{-5 \pm \sqrt{17}}{4}}.$$

**21.** The discriminant is

$$b^2 - 4ac = (-4)^2 - 4(1)(8) = 16 - 32 = -16 < 0.$$

Therefore the equation has $\boxed{\text{no real solutions}}$.

**22.** The discriminant is

$$(-12)^2 - 4(4)(9) = 144 - 144 = 0.$$

There is one repeated real solution:

$$x = \frac{-(-12)}{2(4)} = \boxed{\frac{3}{2}}.$$

**23.** A perfect square with constant $16 = 4^2$ is

$$(x + 4)^2 = x^2 + 8x + 16.$$

Since $k > 0$, $\boxed{k = 8}$.

**24.** One real solution requires a zero discriminant:

$$(-6)^2 - 4(1)(m) = 0 \Rightarrow 36 - 4m = 0 \Rightarrow \boxed{m = 9}.$$

## Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can use exponent laws and solve elementary exponential equations exactly.

**25.** Combine exponents when multiplying and subtract when dividing:

$$\frac{x^3x^5}{x^2} = x^{3+5-2} = \boxed{x^6}, \qquad x \neq 0.$$

**26.** Apply the power to every factor:

$$(2a^3b^{-2})^2 = 4a^6b^{-4} = \boxed{\frac{4a^6}{b^4}}, \qquad b \neq 0.$$

**27.** Use the product rule:

$$3^4 \cdot 3^{-2} = 3^{4-2} = 3^2 = \boxed{9}.$$

**28.** Since $32 = 2^5$,

$$2^x = 2^5 \Rightarrow \boxed{x = 5}.$$

**29.** Since $125 = 5^3$,

$$5^{x+1} = 5^3 \Rightarrow x + 1 = 3 \Rightarrow \boxed{x = 2}.$$

**30.** Write both sides with base $2$:

$$4^{2x} = (2^2)^{2x} = 2^{4x}, \qquad 64 = 2^6.$$

Thus $4x = 6$, so $\boxed{x = \frac{3}{2}}$.

# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can interpret exponential expressions and construct growth, decay, and geometric-sequence models.

**31.** Substitute each input into $f(n) = 6(2)^n$:

$$f(0) = 6, qquadf(1) = 12, qquadf(4) = 6(16) = 96.$$

Therefore $\boxed{f(0) = 6,\ f(1) = 12,\ f(4) = 96}$.

**32.** In $y = a(b)^t$, $a$ is the initial value and $b$ is the factor. Here

$$\boxed{a = 450}, \qquad \boxed{b = 1.08}.$$

Since $1.08 = 1 + 0.08$, this is $\boxed{\text{8\% growth}}$.

**33.** Here the initial value is $\boxed{1200}$ and the factor is $\boxed{0.85}$. Since $0.85 = 1 - 0.15$, this is $\boxed{\text{15\% decay}}$.

**34.** A $6\%$ increase gives a factor of $1 + 0.06 = 1.06$:

$$\boxed{A(t) = 250(1.06)^t}.$$

**35.** A $12\%$ decrease gives a factor of $1 - 0.12 = 0.88$:

$$\boxed{V(t) = 900(0.88)^t}.$$

**36.** Each term is multiplied by $3$. Thus

$$\boxed{a_n = 5(3)^{n-1}}.$$

The next three terms are $45(3) = 135$, $135(3) = 405$, and $405(3) = 1215$:

$$\boxed{135,\ 405,\ 1215}.$$

# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can derive quadratic and exponential rules from values and compare models algebraically.

**37.** The output ratio is constant:

$$\frac{12}{4} = \frac{36}{12} = \frac{108}{36} = 3.$$

The initial value is $4$, so the rule is $\boxed{y = 4(3)^x}$.

**38.** The first differences are $5, 7, 9$ and the second differences are $2, 2$, so $a = 1$ in $y = ax^2 + bx + c$. Since $y(0) = 7$, $c = 7$. Using $y(1) = 12$:

$$1 + b + 7 = 12 \Rightarrow b = 4.$$

Therefore $\boxed{y = x^2 + 4x + 7}$.

**39.** The first differences are $4, 6, 8, 10$, so the constant second difference is $2$ and $a = 1$. Also $c = 2$. Using $y(1) = 6$:

$$1 + b + 2 = 6 \Rightarrow b = 3.$$

Thus $\boxed{y = x^2 + 3x + 2}$.

**40.** Evaluate both rules at $n = 4$:

$$L(4) = 10 + 6(4) = 34, \qquad Q(4) = 4^2 + 3 = 19.$$

Therefore $\boxed{L(4) \text{ is larger by } 15}$.

**41.** At $t = 5$:

$$Q(5) = 5^2 + 8 = 33, \qquad E(5) = 2^6 = 64.$$

Hence $\boxed{E(5) \text{ is larger by } 31}$.

**42.** Test positive integers in order:

| $n$ | 1 | 2 | 3 |
|---|---|---|---|
| $3^n$ | 3 | 9 | 27 |
| $n^2 + 10$ | 11 | 14 | 19 |

The inequality first becomes true at $\boxed{n = 3}$.

# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can build and solve quadratic and exponential models in realistic contexts.

**43.** Let the width be $x$ and the length be $x + 7$:

$$x(x + 7) = 60 \Rightarrow x^2 + 7x - 60 = 0.$$

Factoring gives $(x + 12)(x - 5) = 0$. Reject the negative measurement, so the dimensions are $\boxed{5 \text{ units by } 12 \text{ units}}$.

**44.** Let the smaller integer be $n$:

$$n(n + 1) = 156 \Rightarrow n^2 + n - 156 = 0.$$

Since $(n - 12)(n + 13) = 0$, the positive integers are $\boxed{12 \text{ and } 13}$.

**45.** Set the height equal to zero:

$$-16t^2 + 64t + 80 = 0.$$

Divide by $-16$ and factor:

$$t^2 - 4t - 5 = (t - 5)(t + 1) = 0.$$

Time cannot be negative, so the object reaches the ground after $\boxed{5 \text{ seconds}}$.

**46.** Set the revenue equal to $400$:

$$x(60 - 2x) = 400 \Rightarrow x^2 - 30x + 200 = 0.$$

Since $(x - 10)(x - 20) = 0$, $\boxed{x = 10 \text{ or } x = 20}$.

**47.** Use compound growth:

$$A = 1200(1.05)^4 = 1200(1.21550625) = 1458.6075.$$

Rounded to the nearest cent, the balance is $\boxed{\$1{,}458.61}$.

**48.** A $25\%$ hourly decrease gives a factor of $0.75$:

$$M = 80(0.75)^5 = 18.984375.$$

Thus approximately $\boxed{18.98 \text{ mg}}$ remains.

# Answer Key: Chapter 10 - Bridging Quadratic and Exponential Functions

**Objective:** I can compare long-term models and solve a quadratic equation hidden inside an exponential equation.

**49.** Evaluate both plans at $t = 10$:

$$A(10) = 500(1.04)^{10} \approx 740.12,$$
$$B(10) = 400(1.08)^{10} \approx 863.57.$$

Thus $\boxed{\text{Plan B is larger by approximately } \$123.45}$.

**50.** Let $u = 2^x$, so $u > 0$ and $2^{2x} = u^2$:

$$u^2 - 10u + 16 = 0 \Rightarrow (u-2)(u-8) = 0.$$

Thus $2^x = 2$ or $2^x = 8$, giving

$$\boxed{x = 1 \text{ or } x = 3}.$$

**Key review principles:** distinguish quadratic and exponential structure; preserve exact radical values; check the discriminant before interpreting real solutions; translate percent change into the correct multiplier; and reject values that violate a context.