Answer Key: Chapter 11 - Unlocking Radical Expressions and Triangles

Objective: I can simplify square roots, cube roots, and radical expressions by extracting perfect powers.

Variables are nonnegative unless otherwise stated.

1. Extract the greatest perfect-square factor:

$$\sqrt{72} = \sqrt{36 \cdot 2} = \boxed{6\sqrt{2}}.$$

2. Since $180 = 36 \cdot 5$,

$$\sqrt{180} = \sqrt{36 \cdot 5} = \boxed{6\sqrt{5}}.$$

3. Use $48x^3 = 16x^2(3x)$:

$$\sqrt{48x^3} = \sqrt{16x^2}\sqrt{3x} = \boxed{4x\sqrt{3x}}.$$

4. Extract the perfect cube 27:

$$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \boxed{3\sqrt[3]{2}}.$$

5. Use $50a^2b^3 = 25a^2b^2(2b)$:

$$\sqrt{50a^2b^3} = \boxed{5ab\sqrt{2b}}.$$

6. Combine the radicals and simplify:

$$\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = \boxed{4}.$$

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Objective: I can add, subtract, multiply, and square radical expressions accurately.

7. Combine like radicals:

$$3\sqrt{5} + 7\sqrt{5} - 2\sqrt{5} = (3 + 7 - 2)\sqrt{5} = \boxed{8\sqrt{5}}.$$

8. Simplify before combining:

$$4\sqrt{12} - \sqrt{27} = 4(2\sqrt{3}) - 3\sqrt{3} = \boxed{5\sqrt{3}}.$$

9. Since $\sqrt{8} = 2\sqrt{2}$ and $\sqrt{18} = 3\sqrt{2}$,

$$2\sqrt{8} + 3\sqrt{18} = 4\sqrt{2} + 9\sqrt{2} = \boxed{13\sqrt{2}}.$$

10. Distribute and simplify:

$$\sqrt{6}(\sqrt{15} - 2\sqrt{6}) = \sqrt{90} - 12 = \boxed{3\sqrt{10} - 12}.$$

11. Use the difference-of-squares identity:

$$(\sqrt{7} + 3)(\sqrt{7} - 3) = (\sqrt{7})^2 - 3^2 = 7 - 9 = \boxed{-2}.$$

12. Square the binomial:

$$\begin{aligned}(2\sqrt{3} + \sqrt{5})^2 &= (2\sqrt{3})^2 + 2(2\sqrt{3})(\sqrt{5}) + (\sqrt{5})^2 \\ &= 12 + 4\sqrt{15} + 5 = \boxed{17 + 4\sqrt{15}}.\end{aligned}$$

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Objective: I can rationalize denominators and simplify radical quotients completely.

13. Multiply numerator and denominator by $\sqrt{3}$:

$$\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{5\sqrt{3}}{3}}.$$

14. Multiply by $\sqrt{5}/\sqrt{5}$:

$$\frac{7}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{7\sqrt{5}}{10}}.$$

15. Use the conjugate $\sqrt{6} - \sqrt{2}$:

$$\frac{3}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \boxed{\frac{3(\sqrt{6}-\sqrt{2})}{4}}.$$

16. Use the conjugate $3 + \sqrt{5}$:

$$\frac{4}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{4(3+\sqrt{5})}{9-5} = \boxed{3+\sqrt{5}}.$$

17. Since $\sqrt{8} = 2\sqrt{2}$,

$$\frac{6\sqrt{2}}{3\sqrt{8}} = \frac{6\sqrt{2}}{3(2\sqrt{2})} = \boxed{1}.$$

18. Simplify and rationalize:

$$\frac{\sqrt{12}}{\sqrt{18}} = \frac{2\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}}{3}}.$$

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Objective: I can solve radical equations and reject extraneous solutions by substitution.

19. Square both sides:

$$\sqrt{x+5} = 6 \Rightarrow x+5 = 36 \Rightarrow \boxed{x = 31}.$$

The value checks in the original equation.

20. The right side requires $x \geq 2$. Squaring gives

$$2x - 1 = (x-2)^2 \Rightarrow x^2 - 6x + 5 = 0.$$

The candidates are 1 and 5. Only 5 satisfies the domain and the original equation, so $\boxed{x = 5}$.

21. Isolate the radical, so $x \geq 1$:

$$\sqrt{x+9} = x - 1.$$

Squaring gives

$$x + 9 = (x-1)^2 \Rightarrow x^2 - 3x - 8 = 0.$$

Thus $x = (3 \pm \sqrt{41})/2$. Only the positive candidate checks, so

$$\boxed{x = \frac{3+\sqrt{41}}{2}}.$$

22. Square both sides:

$$3x + 4 = (x+2)^2 \Rightarrow x^2 + x = 0 \Rightarrow x(x+1) = 0.$$

Both candidates check, giving $\boxed{x = -1 \text{ or } x = 0}$.

23. The principal square-root function is one-to-one on nonnegative inputs:

$$\sqrt{x+1} = \sqrt{2x-5} \Rightarrow x + 1 = 2x - 5.$$

Hence $\boxed{x = 6}$, which checks.

24. The domain is $x \geq 1$. Squaring once gives

$$2x + 1 + 2\sqrt{(x+2)(x-1)} = 9,$$

so $\sqrt{(x+2)(x-1)} = 4 - x$. Squaring again:

$$(x+2)(x-1) = (4-x)^2 \Rightarrow 9x = 18.$$

Thus $\boxed{x = 2}$, and $\sqrt{4} + \sqrt{1} = 3$.

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Objective: I can apply the Pythagorean Theorem and classify triangles from their side lengths.

25. For legs 6 and 8,

$$c = \sqrt{6^2 + 8^2} = \sqrt{100} = \boxed{10}.$$

26. Let the missing leg be b:

$$b = \sqrt{13^2 - 5^2} = \sqrt{144} = \boxed{12}.$$

27. For legs 7 and 11,

$$c = \sqrt{7^2 + 11^2} = \sqrt{170} = \boxed{\sqrt{170}}.$$

28. Let the missing leg be b:

$$b = \sqrt{20^2 - 12^2} = \sqrt{256} = \boxed{16}.$$

29. Using the largest side as c:

$$8^2 + 15^2 = 64 + 225 = 289 = 17^2.$$

Therefore the side lengths form a $\boxed{\text{right triangle}}$.

30. Compare using the largest side 12:

$$7^2 + 9^2 = 130 < 144 = 12^2.$$

Therefore the triangle is $\boxed{\text{obtuse}}$.

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Objective: I can use right-triangle structure, square relationships, and similarity to determine unknown lengths.

31. Use the Pythagorean Theorem:

$$x^2 + (x+7)^2 = 17^2.$$

This becomes $x^2 + 7x - 120 = 0 = (x+15)(x-8)$. Reject $x = -15$, so the legs are $\boxed{\text{8 and 15}}$.

32. The altitude bisects the base into two segments of length 5:

$$h = \sqrt{13^2 - 5^2} = \sqrt{144} = \boxed{12}.$$

33. The rectangle's diagonal is the hypotenuse:

$$d = \sqrt{9^2 + 12^2} = \sqrt{225} = \boxed{15}.$$

34. The side length is $s = \sqrt{98} = 7\sqrt{2}$. The diagonal is

$$d = s\sqrt{2} = (7\sqrt{2})(\sqrt{2}) = \boxed{14}.$$

35. The scale factor from hypotenuse 10 to hypotenuse 25 is $25/10 = 2.5$:

$$6(2.5) = 15, \qquad 8(2.5) = 20.$$

The larger triangle's legs are $\boxed{\text{15 and 20}}$.

36. The scale factor from 4 to 10 is $10/4 = 2.5$. Therefore

$$x = 6(2.5) = \boxed{15}.$$

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Objective: I can model distances and measurements with right triangles and exact radicals.

37. The ladder is the hypotenuse:

$$h = \sqrt{15^2 - 9^2} = \sqrt{144} = \boxed{12\text{ ft}}.$$

38. The ramp length is

$$\sqrt{12^2 + 5^2} = \sqrt{169} = \boxed{13\text{ ft}}.$$

39. The wire length is

$$\sqrt{24^2 + 10^2} = \sqrt{676} = \boxed{26\text{ ft}}.$$

40. The rectangular screen's diagonal is

$$\sqrt{15^2 + 8^2} = \sqrt{289} = \boxed{17\text{ in.}}.$$

41. The two travel directions are perpendicular:

$$d = \sqrt{9^2 + 12^2} = \sqrt{225} = \boxed{15\text{ km}}.$$

42. A baseball diamond is a square of side 90 ft. Its diagonal is

$$d = 90\sqrt{2} \approx \boxed{127.3\text{ ft}}.$$

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Objective: I can combine radical simplification and triangle reasoning in multi-step problems.

43. If a square has area 200, then $s^2 = 200$:

$$s = \sqrt{200} = \boxed{10\sqrt{2}\text{ units}}.$$

44. A face of the cube is a square with side 6:

$$d = \sqrt{6^2 + 6^2} = \sqrt{72} = \boxed{6\sqrt{2}\text{ units}}.$$

45. The altitude bisects the side 12 into two lengths of 6:

$$h = \sqrt{12^2 - 6^2} = \sqrt{108} = 6\sqrt{3}.$$

Therefore

$$A = \frac{1}{2}(12)(6\sqrt{3}) = \boxed{36\sqrt{3}\text{ square units}}.$$

46. The altitude bisects the base 12:

$$h = \sqrt{10^2 - 6^2} = 8.$$

Thus $A = \frac{1}{2}(12)(8) = \boxed{48\text{ square units}}$.

47. The direct distance is

$$\sqrt{20^2 + 21^2} = \sqrt{841} = 29.$$

The two-road route is $20 + 21 = 41$, so the direct path saves $\boxed{12\text{ km}}$.

48. The rule $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ is not generally valid. For $a = 9$ and $b = 16$:

$$\sqrt{9+16} = 5, \qquad \sqrt{9} + \sqrt{16} = 3 + 4 = 7.$$

Since $5 \neq 7$, the student's claim is $\boxed{\text{false}}$.

Answer Key: Chapter 11 - Unlocking Radical Expressions and Triangles

Objective: I can recognize a hidden perfect square and solve a two-radical equation exactly.

49. Assume $\sqrt{7+4\sqrt{3}} = \sqrt{a} + \sqrt{b}$. Squaring requires

$$a + b = 7, \qquad ab = 12.$$

The values are $a = 4$ and $b = 3$. Since the principal square root is nonnegative,

$$\boxed{\sqrt{7+4\sqrt{3}} = 2 + \sqrt{3}}.$$

50. The domain is $-4 \le x \le 9$. Square once:

$$13 + 2\sqrt{(x+4)(9-x)} = 25,$$

so $\sqrt{(x+4)(9-x)} = 6$. Squaring again gives

$$(x+4)(9-x) = 36 \Rightarrow x(5-x) = 0.$$

Both candidates check in the original equation, so $\boxed{x = 0 \text{ or } x = 5}$.

Key review principles: extract perfect powers before combining radicals; combine only like radicals; use conjugates to rationalize binomial denominators; check every radical-equation candidate; and identify the hypotenuse before applying the Pythagorean Theorem.