

Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can identify excluded values, evaluate rational expressions, and simplify monomial quotients.

For Questions 1-6, state every restriction from the original expression.

1. State the excluded value for $\frac{5}{x-3}$.	2. State the excluded values for $\frac{x+2}{x^2-9}$.
3. Simplify $\frac{18x^5y^3}{24x^2y}$.	4. Simplify $\frac{-21a^4b^2}{7ab^5}$.
5. For $R(x) = \frac{2x+1}{x-4}$, find $R(6)$.	6. Find all values that make $\frac{3x-12}{x^2-16}$ undefined.

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Objective: I can factor and simplify rational expressions while preserving all original restrictions.

Factor completely before canceling. State restrictions from the original denominator.

7. Simplify $\frac{x^2 - 9}{x^2 + 5x + 6}$.	8. Simplify $\frac{x^2 - 4x}{x^2 - 16}$.
9. Simplify $\frac{2x^2 + 7x + 3}{2x^2 - 5x - 3}$.	10. Simplify $\frac{x^2 - 6x + 9}{x^2 - 9}$.
11. Simplify $\frac{4a^2 - 25}{2a^2 + 7a + 5}$.	12. Simplify $\frac{x^3 - 4x}{x^2 + 4x + 4}$.

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Objective: I can multiply and divide rational expressions and state every restriction.

For division, also exclude values that make the divisor equal to zero.

13. Simplify $\frac{x}{3} \cdot \frac{9}{x^2}$.	14. Simplify $\frac{x^2 - 4}{x^2 + 3x + 2} \cdot \frac{x + 1}{x - 2}$.
15. Simplify $\frac{3x}{x^2 - 9} \cdot \frac{x + 3}{6x^2}$.	16. Simplify $\frac{a^2 - 9}{a^2 - 6a + 9} \div \frac{a + 3}{a - 3}$.
17. Simplify $\frac{x^2 - 1}{x^2 + 2x + 1} \div \frac{x - 1}{x + 1}$.	18. Simplify $\frac{2x^2}{x^2 - 25} \cdot \frac{x^2 - 10x + 25}{4x}$.

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Objective: I can add and subtract rational expressions using least common denominators.

Identify the LCD, combine numerators, and simplify the result.

19. Simplify $\frac{3}{x} + \frac{5}{x}$.	20. Simplify $\frac{7}{2a} - \frac{1}{3a}$.
21. Simplify $\frac{2}{x+1} + \frac{3}{x-1}$.	22. Simplify $\frac{4}{x} - \frac{2}{x+3}$.
23. Simplify $\frac{x}{x-2} - \frac{2}{x+2}$.	24. Simplify $\frac{3}{x^2-4} + \frac{1}{x+2}$.

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Objective: I can simplify complex fractions and solve proportions with algebraic precision.

Clear the complex fraction with an LCD or multiply by a reciprocal.

25. Simplify $\frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{xy}}$.	26. Simplify $\frac{\frac{2}{x}}{\frac{4}{x^2}}$.
27. Simplify $\frac{\frac{x}{x-1} - 1}{\frac{1}{x-1}}$.	28. Simplify $\frac{1}{1 + \frac{1}{x}}$.
29. Solve $\frac{3}{5} = \frac{x}{20}$.	30. Solve $\frac{x-2}{6} = \frac{x+4}{9}$.

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Objective: I can solve rational equations and distinguish valid solutions from excluded values.

List restrictions first, clear denominators, solve, and check.

31. Solve $\frac{5}{x} = 2$.	32. Solve $\frac{3}{x-1} = x$.
33. Solve $\frac{2}{x} + \frac{3}{x+1} = 1$.	34. Solve $\frac{1}{x-2} + \frac{1}{x+2} = \frac{x}{x^2-4}$.
35. Solve $\frac{x+1}{x-3} = 2$.	36. Solve $\frac{x^2-9}{x-3} = 6$. Explain the final result.

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Objective: I can translate rate, work, variation, and formula situations into rational equations.

Define the variable, write the rational equation, and interpret units.

37. A car travels 180 miles in 3 hours at a constant rate. Use $t = d/r$ to find the rate.	38. One pipe fills a tank in 6 hours and another in 3 hours. How long do they take together?
39. Two workers finish a job together in 4 hours. One alone needs 6 hours. How long does the other need alone?	40. A driver travels 60 miles at 30 mph and returns 60 miles at 60 mph. Find the average speed.
41. y varies inversely with x. If $y = 12$ when $x = 5$, find y when $x = 8$.	42. A solution contains 300 mg of medicine. What volume gives a concentration of 20 mg/mL?
43. Solve $\frac{1}{R} = \frac{1}{a} + \frac{1}{b}$ for R. State restrictions.	44. Solve $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ for q. State restrictions.

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Objective: I can analyze rational formulas, algebraic errors, parameters, and challenging equations.

Show all restrictions and a complete algebraic justification.

45. For $f(x) = \frac{1}{x}$, simplify $\frac{f(x+h) - f(x)}{h}$. State restrictions.	46. A student cancels x in $\frac{x+5}{x}$ and writes 5. Explain the error and simplify correctly.
47. Find k so that $\frac{x^2 - k^2}{x - k}$ simplifies to $x+7$ for every allowed x.	48. Find k so $\frac{1}{x-2} = \frac{k}{x-2}$ is true for every allowed x.
49. Honors challenge: Solve $\frac{1}{x-1} + \frac{1}{x+1} = \frac{3}{4}$.	50. Honors challenge: Simplify $\frac{\frac{x}{x-1} - \frac{x-1}{x}}{\frac{1}{x(x-1)}}$.

Other topics for review:

- Restrictions are determined before any factors are canceled.
- Division by a rational expression also requires the divisor to be nonzero.
- The LCD clears denominators but does not automatically validate a solution.
- Average speed equals total distance divided by total time, not the mean of two speeds.
- Cancellation is valid only for common factors of an entire numerator and denominator.