Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can identify excluded values, evaluate rational expressions, and simplify monomial quotients.

Every excluded value comes from the original denominator.

1. The denominator cannot equal zero:

$$x - 3 \neq 0 \Rightarrow \boxed{x \neq 3}.$$

2. Factor the denominator:

$$x^2 - 9 = (x-3)(x+3).$$

Thus the excluded values are $\boxed{x \neq -3,\ 3}$.

3. Reduce coefficients and subtract exponents:

$$\frac{18x^5y^3}{24x^2y} = \boxed{\frac{3x^3y^2}{4}}, \qquad x \neq 0,\ y \neq 0.$$

4. Divide coefficients and subtract exponents:

$$\frac{-21a^4b^2}{7ab^5} = \boxed{-\frac{3a^3}{b^3}}, \qquad a \neq 0,\ b \neq 0.$$

5. Substitute $x = 6$:

$$R(6) = \frac{2(6)+1}{6-4} = \boxed{\frac{13}{2}}.$$

6. Set the denominator equal to zero:

$$x^2 - 16 = (x-4)(x+4) = 0.$$

The expression is undefined at $\boxed{x = -4 \text{ and } x = 4}$.

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can factor and simplify rational expressions while preserving all original restrictions.

7. Factor numerator and denominator:

$$\frac{(x-3)(x+3)}{(x+2)(x+3)} = \boxed{\frac{x-3}{x+2}}, \qquad x \neq -3, -2.$$

8. Factor and cancel the common factor:

$$\frac{x(x-4)}{(x-4)(x+4)} = \boxed{\frac{x}{x+4}}, \qquad x \neq -4, 4.$$

9. Factor both trinomials:

$$\frac{(2x+1)(x+3)}{(2x+1)(x-3)} = \boxed{\frac{x+3}{x-3}}, \qquad x \neq -\frac{1}{2}, 3.$$

10. Use a perfect square and a difference of squares:

$$\frac{(x-3)^2}{(x-3)(x+3)} = \boxed{\frac{x-3}{x+3}}, \qquad x \neq -3, 3.$$

11. Factor:

$$\frac{(2a-5)(2a+5)}{(2a+5)(a+1)} = \boxed{\frac{2a-5}{a+1}}, \qquad a \neq -\frac{5}{2}, -1.$$

12. Factor completely:

$$\frac{x(x-2)(x+2)}{(x+2)^2} = \boxed{\frac{x(x-2)}{x+2}}, \qquad x \neq -2.$$

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can multiply and divide rational expressions and state every restriction.

13. Multiply and cancel:

$$\frac{x}{3} \cdot \frac{9}{x^2} = \boxed{\frac{3}{x}}, \qquad x \neq 0.$$

14. Factor before multiplying:

$$\frac{(x-2)(x+2)}{(x+1)(x+2)} \cdot \frac{x+1}{x-2} = \boxed{1}.$$

The original restrictions are $\boxed{x \neq -2, -1, 2}$.

15. Factor and cancel common factors:

$$\frac{3x}{(x-3)(x+3)} \cdot \frac{x+3}{6x^2} = \boxed{\frac{1}{2x(x-3)}}.$$

The restrictions are $\boxed{x \neq -3, 0, 3}$.

16. Multiply by the reciprocal:

$$\frac{(a-3)(a+3)}{(a-3)^2} \cdot \frac{a-3}{a+3} = \boxed{1}.$$

Also exclude values that make the divisor zero, so $\boxed{a \neq -3, 3}$.

17. Rewrite division as multiplication by the reciprocal:

$$\frac{(x-1)(x+1)}{(x+1)^2} \cdot \frac{x+1}{x-1} = \boxed{1}.$$

The divisor cannot be zero, so $\boxed{x \neq -1, 1}$.

18. Factor and reduce:

$$\frac{2x^2}{(x-5)(x+5)} \cdot \frac{(x-5)^2}{4x} = \boxed{\frac{x(x-5)}{2(x+5)}}.$$

The restrictions are $\boxed{x \neq -5, 0, 5}$.

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can add and subtract rational expressions using least common denominators.

19. The denominators are already equal:

$$\frac{3}{x}+\frac{5}{x}=\boxed{\frac{8}{x}},\qquad x\neq 0.$$

20. The LCD is $6a$:

$$\frac{7}{2a}-\frac{1}{3a}=\frac{21}{6a}-\frac{2}{6a}=\boxed{\frac{19}{6a}},\qquad a\neq 0.$$

21. Use the LCD $(x+1)(x-1)$:

$$\frac{2(x-1)+3(x+1)}{x^2-1}=\boxed{\frac{5x+1}{x^2-1}},\qquad x\neq -1,1.$$

22. Use the LCD $x(x+3)$:

$$\frac{4(x+3)-2x}{x(x+3)}=\boxed{\frac{2(x+6)}{x(x+3)}},\qquad x\neq -3,0.$$

23. Use the LCD $(x-2)(x+2)$:

$$\frac{x(x+2)-2(x-2)}{x^2-4}=\boxed{\frac{x^2+4}{x^2-4}},\qquad x\neq -2,2.$$

24. Rewrite the second fraction using the LCD x^2-4:

$$\frac{3}{x^2-4}+\frac{x-2}{x^2-4}=\boxed{\frac{x+1}{x^2-4}},\qquad x\neq -2,2.$$

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can simplify complex fractions and solve proportions with algebraic precision.

25. The numerator is $(x+y)/(xy)$:

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}} = \frac{x+y}{xy}\cdot xy = \boxed{x+y}, \qquad x \neq 0,\ y \neq 0.$$

26. Multiply by the reciprocal:

$$\frac{\frac{2}{x}}{\frac{4}{x^2}} = \frac{2}{x}\cdot\frac{x^2}{4} = \boxed{\frac{x}{2}}, \qquad x \neq 0.$$

27. Simplify the numerator first:

$$\frac{x}{x-1} - 1 = \frac{x-(x-1)}{x-1} = \frac{1}{x-1}.$$

Therefore

$$\frac{\frac{x}{x-1}-1}{\frac{1}{x-1}} = \boxed{1}, \qquad x \neq 1.$$

28. Combine the denominator:

$$\frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = \boxed{\frac{x}{x+1}}.$$

The original complex fraction requires $\boxed{x \neq 0, -1}$.

29. Cross-multiply:

$$\frac{3}{5} = \frac{x}{20} \Rightarrow 5x = 60 \Rightarrow \boxed{x = 12}.$$

30. Cross-multiply and solve:

$$\frac{x-2}{6} = \frac{x+4}{9} \Rightarrow 9x - 18 = 6x + 24 \Rightarrow \boxed{x = 14}.$$

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can solve rational equations and distinguish valid solutions from excluded values.

31. Multiply by x, where $x \neq 0$:

$$\frac{5}{x} = 2 \Rightarrow 5 = 2x \Rightarrow \boxed{x = \frac{5}{2}}.$$

32. The restriction is $x \neq 1$. Multiply by $x - 1$:

$$\frac{3}{x-1} = x \Rightarrow 3 = x^2 - x.$$

Thus

$$x = \frac{1 \pm \sqrt{13}}{2}.$$

Both values are allowed, so $\boxed{x = (1 \pm \sqrt{13})/2}$.

33. The restrictions are $x \neq 0, -1$. Multiply by $x(x+1)$:

$$2(x+1) + 3x = x(x+1).$$

Therefore $x^2 - 4x - 2 = 0$, giving

$$\boxed{x = 2 \pm \sqrt{6}}.$$

Both values check.

34. The restrictions are $x \neq -2, 2$. Multiply by $x^2 - 4$:

$$(x+2) + (x-2) = x \Rightarrow 2x = x.$$

Hence $\boxed{x = 0}$, which is allowed.

35. The restriction is $x \neq 3$. Multiply by $x - 3$:

$$x + 1 = 2(x-3) \Rightarrow x + 1 = 2x - 6 \Rightarrow \boxed{x = 7}.$$

36. The original expression requires $x \neq 3$. Factor and simplify:

$$\frac{x^2 - 9}{x - 3} = x + 3.$$

The resulting equation $x + 3 = 6$ gives $x = 3$, but that value is excluded. Therefore the equation has $\boxed{\text{no solution}}$.

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can translate rate, work, average-speed, and inverse-variation situations into rational equations.

37. Use $t = d/r$:

$$\frac{180}{r} = 3 \Rightarrow 180 = 3r \Rightarrow \boxed{r = 60 \text{ mph}}.$$

38. Add the filling rates:

$$\frac{1}{6} + \frac{1}{3} = \frac{1}{2} \text{ tank per hour.}$$

Thus the filling time is $1/(1/2) = \boxed{2 \text{ hours}}$.

39. Let the second worker's rate be $1/t$:

$$\frac{1}{6} + \frac{1}{t} = \frac{1}{4}.$$

Then $1/t = 1/12$, so $\boxed{t = 12 \text{ hours}}$.

40. The total distance is 120 miles. The total time is

$$\frac{60}{30} + \frac{60}{60} = 2 + 1 = 3 \text{ hours.}$$

Therefore the average speed is $120/3 = \boxed{40 \text{ mph}}$.

41. For inverse variation, $y = k/x$. Since $12 = k/5$, $k = 60$. Therefore

$$y = \frac{60}{8} = \boxed{7.5}.$$

42. Let v be the volume in milliliters:

$$\frac{300}{v} = 20 \Rightarrow 300 = 20v \Rightarrow \boxed{v = 15 \text{ mL}}.$$

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can rearrange rational formulas, analyze algebraic errors, and reason about parameters.

43. Combine the right side:

$$\frac{1}{R} = \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}.$$

Taking reciprocals gives

$$\boxed{R = \frac{ab}{a+b}},$$

where $a \neq 0$, $b \neq 0$, and $a + b \neq 0$.

44. Start with

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}.$$

Then $1/q = 1/f - 1/p = (p - f)/(fp)$, so

$$\boxed{q = \frac{fp}{p-f}},$$

with $f \neq 0$, $p \neq 0$, and $p \neq f$.

45. For $f(x) = 1/x$,

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \frac{-h}{x(x+h)} \cdot \frac{1}{h} = \boxed{-\frac{1}{x(x+h)}}.\end{aligned}$$

The restrictions are $\boxed{x \neq 0,\ h \neq 0,\ x + h \neq 0}$.

46. Cancellation applies to factors, not terms joined by addition. The correct rewrite is

$$\frac{x+5}{x} = \frac{x}{x} + \frac{5}{x} = \boxed{1 + \frac{5}{x}}, \qquad x \neq 0.$$

Thus the student's answer 5 is incorrect.

47. For $x \neq k$,

$$\frac{x^2 - k^2}{x - k} = \frac{(x-k)(x+k)}{x-k} = x + k.$$

To equal $x + 7$, we need $\boxed{k = 7}$; the original expression then excludes $x = 7$.

48. For allowed values $x \neq 2$, multiply both sides by $x - 2$:

$$1 = k.$$

Therefore the equation is true for every allowed x precisely when $\boxed{k = 1}$.

Answer Key: Chapter 12 - Navigating Rational Expressions and Equations

Objective: I can solve a multi-step rational equation and simplify a complex rational identity.

49. The restrictions are $x \neq -1, 1$. Multiply by $4(x^2 - 1)$:

$$4(x+1) + 4(x-1) = 3(x^2 - 1).$$

Thus

$$3x^2 - 8x - 3 = 0 = (3x+1)(x-3).$$

Both solutions are allowed, so

$$\boxed{x = -\frac{1}{3} \text{ or } x = 3}.$$

50. Combine the numerator:

$$\frac{x}{x-1} - \frac{x-1}{x} = \frac{x^2 - (x-1)^2}{x(x-1)} = \frac{2x-1}{x(x-1)}.$$

Now divide by $1/[x(x-1)]$:

$$\frac{\frac{2x-1}{x(x-1)}}{\frac{1}{x(x-1)}} = \boxed{2x-1}, \qquad x \neq 0, 1.$$

Key review principles: factor before canceling; restrictions come from original denominators and divisors; use the LCD to clear fractions; check proposed solutions against excluded values; and remember that only common factors, never separate terms, may be canceled.