Answer Key: Chapter 13 - Exploring the World of Statistics

Objective: I can distinguish populations, samples, data types, and sampling methods.
Use precise statistical vocabulary and explain possible bias.

1. The population is $\boxed{\text{all 1,200 students at the school}}$. The sample is $\boxed{\text{the 80 surveyed students}}$.

2. The 62% describes the surveyed sample rather than the entire population, so it is a $\boxed{\text{statistic}}$.

3. Shoe size is numerical, so it is $\boxed{\text{quantitative}}$. Favorite subject records a label, so it is $\boxed{\text{categorical}}$.

4. The number of siblings is countable and therefore $\boxed{\text{discrete}}$. Height may take any value in an interval and is therefore $\boxed{\text{continuous}}$.

5. Surveying only basketball-team members is a $\boxed{\text{convenience sample}}$ and is biased. Team members are likely more physically active than the full student body, so the survey would probably $\boxed{\text{overestimate exercise time}}$.

6. Assign every student a unique number from 1 to $1{,}200$, then use a random-number process to select 100 distinct numbers. This is a $\boxed{\text{simple random sample}}$ because every student has an equal chance of selection.

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Objective: I can calculate and interpret measures of center and spread.

7. Add the values and divide by 5:

$$\bar{x} = \frac{6+8+8+10+13}{5} = \frac{45}{5} = \boxed{9}.$$

8. For $6, 8, 8, 10, 13$, the middle value is 8, the repeated value is 8, and

$$13 - 6 = 7.$$

Thus $\boxed{\text{median} = 8}$, $\boxed{\text{mode} = 8}$, and $\boxed{\text{range} = 7}$.

9. Order the data:

$$9, 11, 12, 13, 15, 18.$$

The median is the average of the two middle values:

$$\frac{12+13}{2} = \boxed{12.5}.$$

10. A mean of 18 for 5 values requires a total of $5(18) = 90$. The known values total 72, so the missing value is

$$90 - 72 = \boxed{18}.$$

11. A mean of 8 for 5 values requires a sum of 40:

$$4 + 7 + 9 + 10 + x = 40 \Rightarrow 30 + x = 40.$$

Therefore $\boxed{x = 10}$.

12. Originally,

$$\bar{x} = \frac{32}{5} = 6.4, \qquad \text{median} = 6.$$

After adding 30,

$$\bar{x} = \frac{62}{6} \approx 10.33, \qquad \text{median} = \frac{6+7}{2} = 6.5.$$

The mean changes by about 3.93, but the median changes by only 0.5. Thus the $\boxed{\text{median is more resistant to the outlier}}$.

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Objective: I can find quartiles, interquartile range, five-number summaries, and outliers.
Use the median-of-halves convention, excluding the overall median when necessary.

13. For $2, 4, 5, 7, 9, 10, 12, 15$,

$$Q_1 = \frac{4+5}{2} = 4.5, \qquad M = \frac{7+9}{2} = 8,$$

$$Q_3 = \frac{10+12}{2} = 11, \qquad IQR = 11 - 4.5 = 6.5.$$

Therefore $\boxed{Q_1 = 4.5,\ M = 8,\ Q_3 = 11,\ IQR = 6.5}$.

14. Using the quartiles from Question 13, the five-number summary is

$$\boxed{(2,\ 4.5,\ 8,\ 11,\ 15)}.$$

15. Here $IQR = 22 - 10 = 12$. The fences are

$$10 - 1.5(12) = -8, \qquad 22 + 1.5(12) = 40.$$

Of $5, 12, 18, 22, 41$, only $\boxed{41}$ lies outside the fences.

16. For $3, 5, 6, 8, 9, 11, 12, 20$,

$$Q_1 = 5.5, \quad Q_3 = 11.5, \quad IQR = 6.$$

The fences are -3.5 and 20.5. Since 20 lies within them, it is $\boxed{\text{not an outlier}}$.

17. Adding 4 to every value adds 4 to the mean, median, Q_1, and Q_3. Differences are unchanged, so the range and IQR remain the same. Therefore

$$\boxed{\text{center measures increase by } 4;\ \text{spread measures are unchanged}}.$$

18. Multiplying every value by 3 multiplies measures of center and spread by 3:

$$\text{new median} = 3(7) = \boxed{21}, \qquad \text{new IQR} = 3(4) = \boxed{12}.$$

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Objective: I can calculate and interpret mean absolute deviation, variance, standard deviation, and standard scores.

19. The mean of $2, 4, 6, 8, 10$ is 6. The absolute deviations are $4, 2, 0, 2, 4$:

$$MAD = \frac{4+2+0+2+4}{5} = \boxed{2.4}.$$

20. For $2, 4, 6$, the population mean is 4. Thus

$$\sigma^2 = \frac{(2-4)^2+(4-4)^2+(6-4)^2}{3} = \boxed{\frac{8}{3}}.$$

Therefore

$$\sigma = \sqrt{\frac{8}{3}} = \boxed{\frac{2\sqrt{6}}{3} \approx 1.63}.$$

21. Every value equals the mean, so every deviation is zero. The population standard deviation is $\boxed{0}$.

22. Both sets have mean 10, but Set A has no deviation from its mean. Set B ranges from 6 to 14. Therefore $\boxed{\text{Set B}}$ has greater variability.

23. Use $z = (x - \mu)/\sigma$:

$$z = \frac{78-70}{4} = \boxed{2}.$$

The score is two standard deviations above the mean.

24. Use $x = \mu + z\sigma$:

$$x = 50 + (-1.5)(6) = 50 - 9 = \boxed{41}.$$

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Objective: I can extract measures of center from frequency data and calculate weighted means.

25. The total frequency is 10. The weighted total is

$$1(2) + 2(5) + 3(2) + 4(1) = 22.$$

Thus the mean is $\boxed{22/10 = 2.2}$.

26. There are 10 observations, so use positions 5 and 6. Both positions contain the value 2, giving $\boxed{\text{median} = 2}$.

27. The value 2 has the greatest frequency, 5. Therefore $\boxed{\text{mode} = 2}$.

28. Multiply each category by its weight:

$$0.60(80) + 0.25(90) + 0.15(100) = 48 + 22.5 + 15 = \boxed{85.5\%}.$$

29. Use total points divided by total students:

$$\frac{20(72) + 30(84)}{20 + 30} = \frac{3960}{50} = \boxed{79.2}.$$

30. The total frequency is 10, and

$$70(1) + 80(3) + 90(4) + 100(2) = 870.$$

Therefore the mean is $\boxed{870/10 = 87}$.

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Objective: I can calculate joint, marginal, and conditional relative frequencies and assess association.

31. The joint count for Grade 9 and soccer is 18 out of 60 students:

$$\frac{18}{60} = \boxed{30\%}.$$

32. Among Grade 10 students, 16 of 30 prefer basketball:

$$\frac{16}{30} \approx \boxed{53.3\%}.$$

33. The conditional soccer percentages are

$$\frac{18}{30} = 60\% \quad \text{for Grade 9,} \qquad \frac{14}{30} \approx 46.7\% \quad \text{for Grade 10.}$$

Thus $\boxed{\text{Grade 9 has the higher soccer preference by about } 13.3 \text{ points}}$.

34. The pass rates are

$$\frac{36}{40} = \boxed{90\%} \quad \text{with attendance,} \qquad \frac{24}{40} = \boxed{60\%} \quad \text{without attendance.}$$

35. The conditional pass rates differ by

$$90\% - 60\% = \boxed{30 \text{ percentage points}}.$$

This shows an association between attendance and passing, but does not by itself prove causation.

36. There are 20 students who did not pass out of 80 total:

$$\frac{20}{80} = \boxed{25\%}.$$

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Objective: I can interpret algebraic prediction models, correlation, and residuals without visual displays.

37. The y-values increase by 2 whenever x increases by 1, and $y = 3$ when $x = 1$. The exact rule is

$$\boxed{y = 2x + 1}.$$

38. In $S = 6h + 52$, the slope is 6. The model predicts

$$\boxed{6 \text{ additional score points per study hour}}.$$

39. The intercept is 52. It represents the model's prediction when $h = 0$:

$$\boxed{52 \text{ points with no study hours}}.$$

40. At $h = 5$, the predicted score is

$$6(5) + 52 = 82.$$

Residual $=$ actual $-$ predicted, so

$$83 - 82 = \boxed{1}.$$

41. A correlation coefficient of $r = 0.91$ is close to 1. It indicates a $\boxed{\text{strong positive linear association}}$.

42. $r = -0.18$ indicates a $\boxed{\text{weak negative}}$ linear association. Correlation summarizes association; it does not establish causation.

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Objective: I can evaluate predictions, correct statistical errors, and design representative samples.

43. For $x = 4$,

$$A(4) = 5(4) + 9 = 29, \qquad B(4) = 6(4) + 4 = 28.$$

With actual value 30, the absolute residuals are 1 and 2. Therefore $\boxed{\text{Model A is closer at } x = 4}$.

44. Correcting 42 to 72 increases the total by 30. For 10 observations, the mean increases by

$$\frac{30}{10} = \boxed{3}.$$

45. For 11 ordered observations, the median is the 6th value. Changing only the largest value, while it remains largest, does not change the 6th value. Thus the median is $\boxed{\text{unchanged}}$.

46. The classes have different sizes, so use a weighted mean:

$$\frac{10(80) + 30(90)}{40} = \frac{3500}{40} = \boxed{87.5}.$$

The unweighted answer 85 is incorrect.

47. Because only people who choose to respond are included, the method has $\boxed{\text{voluntary-response bias}}$. People with strong opinions may be overrepresented, so the result may not describe the target population.

48. Use proportional allocation. From a sample of 100, select

$$0.60(100) = \boxed{60} \quad \text{younger-grade students}$$

and

$$0.40(100) = \boxed{40} \quad \text{older-grade students,}$$

randomly within each group.

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Objective: I can reconstruct a data value from summary conditions and transform statistical measures.

49. A mean of 20 for 5 values requires total 100:

$$12 + 18 + 18 + a + 32 = 100.$$

The known total is 80, so $\boxed{a = 20}$. The ordered data remain $12, 18, 18, 20, 32$, satisfying the stated median and mode.

50. For the transformation $y = 2x - 5$, the mean transforms as

$$\mu_y = 2(14) - 5 = \boxed{23}.$$

Adding or subtracting a constant does not change standard deviation, while multiplication by 2 doubles it:

$$\sigma_y = 2(3) = \boxed{6}.$$

Key review principles: match the statistic to the question; distinguish samples from populations; use weighted totals when group sizes differ; apply the $1.5(IQR)$ rule consistently; interpret residual as actual minus predicted; and never infer causation from correlation alone.