Answer Key: Chapter 14 - Embracing the Odds in Probability

Objective: I can calculate probabilities from equally likely outcomes and use complements.

Write each probability as a simplified fraction and, when useful, a decimal or percent.

1. A fair number cube has outcomes 1 through 6. Outcomes greater than 4 are 5 and 6:

$$P(\text{greater than } 4) = \frac{2}{6} = \boxed{\frac{1}{3}}.$$

2. A standard deck has 13 hearts among 52 cards:

$$P(\text{heart}) = \frac{13}{52} = \boxed{\frac{1}{4}}.$$

3. The word ALGEBRA has 7 letters. The vowels are A, E, and A:

$$P(\text{vowel}) = \boxed{\frac{3}{7}}.$$

4. Use the complement rule:

$$P(\text{no rain}) = 1 - P(\text{rain}) = 1 - 0.35 = \boxed{0.65}.$$

5. The sample space is

$$\{HH, HT, TH, TT\}.$$

Exactly one head occurs in HT and TH, so

$$P(\text{exactly one head}) = \frac{2}{4} = \boxed{\frac{1}{2}}.$$

6. Two fair number cubes have $6 \cdot 6 = 36$ ordered outcomes. Only $(1, 1)$ has sum 2, so

$$P(\text{sum } 2) = \boxed{\frac{1}{36}}.$$

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Objective: I can compare experimental and theoretical probability and predict long-run frequency.

7. Experimental probability is successes divided by trials:

$$\frac{48}{80} = \boxed{0.60 = 60\%}.$$

8. Among $1, 2, \ldots, 8$, the multiples of 3 are 3 and 6:

$$P(\text{multiple of } 3) = \frac{2}{8} = \boxed{\frac{1}{4}}.$$

9. The experimental probability is

$$\frac{68}{120} = \frac{17}{30} \approx 0.567.$$

The theoretical probability is 0.5. The experiment is about

$$0.567 - 0.500 = \boxed{0.067}$$

higher.

10. Multiply the number of trials by the probability:

$$300\left(\frac{1}{6}\right) = \boxed{50}.$$

11. The estimated defect probability is

$$\frac{7}{250} = 0.028 = \boxed{2.8\%}.$$

12. Use the estimated rate from Question 11:

$$1800(0.028) = \boxed{50.4}.$$

Thus about $\boxed{50}$ defective items would be expected in a whole-number prediction.

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Objective: I can calculate compound probabilities for independent and dependent events.

13. The coin and number cube are independent:

$$P(H \text{ and even}) = \frac{1}{2} \cdot \frac{3}{6} = \boxed{\frac{1}{4}}.$$

14. Replacement keeps the events independent:

$$P(\text{two aces}) = \frac{4}{52} \cdot \frac{4}{52} = \boxed{\frac{1}{169}}.$$

15. Without replacement, only 3 aces remain among 51 cards:

$$P(\text{two aces}) = \frac{4}{52} \cdot \frac{3}{51} = \boxed{\frac{1}{221}}.$$

16. After one red item is selected, 4 red remain among 7 items:

$$P(RR) = \frac{5}{8} \cdot \frac{4}{7} = \boxed{\frac{5}{14}}.$$

17. For red followed by blue without replacement:

$$P(RB) = \frac{5}{8} \cdot \frac{3}{7} = \boxed{\frac{15}{56}}.$$

18. Use the complement of no heads:

$$P(\text{at least one head}) = 1 - P(TTT) = 1 - \left(\frac{1}{2}\right)^3 = \boxed{\frac{7}{8}}.$$

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Objective: I can use the Fundamental Counting Principle, permutations, and combinations.

19. Multiply the independent choices:

$$4 \cdot 3 \cdot 2 = \boxed{24 \text{ outfits}}.$$

20. Each of the 5 PIN positions has 10 choices:

$$10^5 = \boxed{100{,}000 \text{ PINs}}.$$

21. For four distinct letters with no repetition:

$$26 \cdot 25 \cdot 24 \cdot 23 = \boxed{358{,}800 \text{ codes}}.$$

22. All 6 books are distinct, so the number of arrangements is

$$6! = \boxed{720}.$$

23. Order does not matter for a committee:

$$\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = \boxed{120}.$$

24. First, second, and third places are different roles, so order matters:

$${}_8P_3 = 8 \cdot 7 \cdot 6 = \boxed{336}.$$

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Objective: I can calculate joint, marginal, conditional, and union probabilities from a two-way table.

25. There are 33 bus riders among 80 students:

$$P(B) = \boxed{\frac{33}{80}}.$$

26. The joint count for Grade 10 and car is 20:

$$P(G10 \cap C) = \frac{20}{80} = \boxed{\frac{1}{4}}.$$

27. Condition on the 40 Grade 10 students:

$$P(C \mid G10) = \frac{20}{40} = \boxed{\frac{1}{2}}.$$

28. Condition on the 30 car riders:

$$P(G10 \mid C) = \frac{20}{30} = \boxed{\frac{2}{3}}.$$

29. If grade and bus use were independent, $P(B \mid G9)$ would equal $P(B)$. But

$$P(B \mid G9) = \frac{18}{40} = 0.45, \qquad P(B) = \frac{33}{80} = 0.4125.$$

Since these differ, the events are $\boxed{\text{not independent}}$.

30. Use inclusion-exclusion:

$$P(W \cup G9) = \frac{17 + 40 - 12}{80} = \frac{45}{80} = \boxed{\frac{9}{16}}.$$

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Objective: I can apply union, intersection, complement, and independence rules.

31. Use inclusion-exclusion:

$$P(A \cup B) = 0.45 + 0.30 - 0.12 = \boxed{0.63}.$$

32. The complement of $A \cup B$ is neither event:

$$P(\text{neither}) = 1 - 0.63 = \boxed{0.37}.$$

33. Mutually exclusive events have no overlap:

$$P(A \cup B) = 0.25 + 0.40 = \boxed{0.65}.$$

34. Rearrange inclusion-exclusion:

$$P(A \cap B) = 0.48 + 0.39 - 0.72 = \boxed{0.15}.$$

35. For independent events,

$$P(A \cap B) = 0.60(0.35) = \boxed{0.21}.$$

Then

$$P(A \cup B) = 0.60 + 0.35 - 0.21 = \boxed{0.74}.$$

36. For mutually exclusive events, $P(A \cap B) = 0$. Independence would require $P(A \cap B) = P(A)P(B)$, which is positive when both probabilities are nonzero. Therefore such events are $\boxed{\text{not independent}}$.

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Objective: I can calculate expected value, judge fairness, and estimate applied risk.

37. Use gains as positive and losses as negative:

$$E = 0.25(8) + 0.75(-2) = 2 - 1.5 = \boxed{\$0.50}.$$

38. The expected prize is

$$0.25(12) + 0.75(0) = \$3.$$

After the 3-dollar cost, expected net value is $\boxed{\$0}$, so the game is $\boxed{\text{fair}}$.

39. The expected prize per ticket is $500/200 = \$2.50$. After the 4 cost,

$$E = 2.50 - 4.00 = \boxed{-\$1.50}.$$

40. Multiply each value by its probability:

$$E(X) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = \boxed{1.7}.$$

41. All probabilities are between 0 and 1, and

$$0.20 + 0.35 + 0.25 + 0.20 = 1.$$

Therefore the model is $\boxed{\text{valid}}$.

42. Although all entries are nonnegative,

$$0.40 + 0.35 + 0.30 = 1.05 \neq 1.$$

Therefore the model is $\boxed{\text{invalid}}$.

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Objective: I can solve multi-step probability applications and identify common reasoning errors.

43. The probability of no rain on one day is 0.80. Assuming independence,

$$P(\text{no rain both days}) = 0.80^2 = \boxed{0.64}.$$

44. The probability that none of 4 items is defective is 0.97^4. Thus

$$P(\text{at least one defective}) = 1 - 0.97^4 \approx \boxed{0.1147 = 11.47\%}.$$

45. June and July are 2 of 12 equally likely months:

$$P(\text{June or July}) = \frac{2}{12} = \boxed{\frac{1}{6}}.$$

46. Out of $10{,}000$ people, 200 have the condition and $0.90(200) = 180$ test positive. Of $9{,}800$ without it, $0.05(9800) = 490$ test positive. Therefore

$$P(\text{condition} \mid \text{positive}) = \frac{180}{180 + 490} = \boxed{\frac{18}{67} \approx 26.9\%}.$$

47. Adding $P(A)$ and $P(B)$ counts the overlap twice. The correct rule is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Thus

$$0.40 + 0.50 - 0.20 = \boxed{0.70},$$

not 0.90.

48. Without replacement, the second probability changes:

$$P(GG) = \frac{6}{10} \cdot \frac{5}{9} = \boxed{\frac{1}{3}}.$$

The student's $(6/10)^2$ incorrectly treats the events as independent.

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Objective: I can use binomial counting and reverse conditional reasoning in honors-level problems.

49. Choose which 2 of the 4 trials are successes, then multiply probabilities:

$$P(X=2)=\binom{4}{2}(0.3)^2(0.7)^2=6(0.09)(0.49)=\boxed{0.2646}.$$

50. Let the two boxes be equally likely. The probability of selecting Box A and then red is

$$\frac{1}{2}\cdot\frac{3}{4}=\frac{3}{8}.$$

The total probability of red is

$$\frac{1}{2}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{2}.$$

Therefore

$$P(A\mid R)=\frac{3/8}{1/2}=\boxed{\frac{3}{4}}.$$

Key review principles: use complements for “at least one” events; distinguish replacement from no replacement; multiply only with the correct conditional probabilities; subtract overlap in union rules; use combinations when order does not matter; and interpret expected value as a long-run average.