

Answer Key: Chapter 1 - The Foundation of Algebra

A. Algebraic Language

Objective: Identify the parts of algebraic expressions and translate accurately between words and symbols.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Expression structure term = coefficient $\times$ variable part like terms have identical variable parts	Core conventions grouping $\rightarrow$ exponents $\rightarrow$ multiply/divide $\rightarrow$ add/subtract $a(b+c)=ab+ac$
1. Variable: x ; coefficient: 7 ; constant term: -12 . The terms are connected by subtraction (equivalently, addition of -12).	2. (a) expression; (b) equation; (c) expression; (d) equation. An equation contains an equals sign asserting that two expressions have the same value.
3. The terms are $5x^2$, $-3x$, and 8. Their coefficients are 5, -3, and 8 (the constant coefficient is 8).	4. Three times n is $3n$; eight less than that is $3n-8$.
5. One correct phrase is: four times the quantity x plus 7, divided by 3 .	6. The difference of x and 5 is $x-5$, and its square is $(x-5)^2$. The student's expression squares only x.

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B. Evaluating Expressions

Objective: Substitute signed values correctly and interpret formulas for geometric quantities.

7. $4(-2)-3=-8-3=-11$.

8. $2(-3)^2-3(4)=2(9)-12=6$.

9. $(5+2)^2/(5-2)=49/3$. Answer: **49/3**.

10. $(-2)(3)(-4)+3^2=24+9=33$.

11. $3|2(-1)-5|-4=3|-7|-4=21-4=17$.

12. $P=2(2w+3)+2w=6w+6$. At $w=6$, $P=36+6=42$ units.

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C. Order of Operations

Objective: Apply grouping, exponents, and left-to-right conventions with precision.

13. Parentheses first: $18 - 3(6) = 18 - 18 = \mathbf{0}$.

14. Multiplication and division have equal rank, so work left to right: $24 \div 3 = 8$, then $8 \cdot 2 = 16$. Thus $6 + 16 = \mathbf{22}$.

15. Inside: $8 - 5 = 3$, then $3 + 3 = 6$. Thus $25 - 2(6) = 25 - 12 = \mathbf{13}$.

16. The bracketed denominator is $2(4) = 8$, so $48 \div 8 = \mathbf{6}$. The brackets specify that the entire product is the divisor.

17. Exponent before the leading negative: $-3^2 = -9$, while $(-3)^2 = 9$. Sum: **0**.

18. One valid placement is $\mathbf{8 + 4(3 - 2) = 8 + 4 = 12}$, not 34. To obtain 34, place parentheses as $\mathbf{(8 + 4) \cdot 3 - 2 = 34}$.

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D. Properties of Real-Number Operations

Objective: Recognize and apply commutative, associative, distributive, and factoring structures.

19. Associative property of addition; the grouping changes, but the order does not.

20. Distributive property.

21. $13+8=8+13$ by the **commutative property of addition**.

22. $-4(2x-7)=-8x+28$. Answer: **$-8x+28$** .

23. The GCF is 6: $18x+24=6(3x+4)$.

24. The commutative property gives $ab=ba$ and $ac=ca$. Therefore $ab+ac=ba+ca$; distributing on either side produces equivalent expressions.

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E. Simplifying Expressions

Objective: Distribute, remove grouping, and combine only like terms.

25. $(7x-3x)+(4+9)=4x+13$.

26. $(5a+3a)+(-2b+7b)-6=8a+5b-6$.

27. $8x-12-5x+9=3x-3$.

28. $-6y-15+4y-4=-2y-19$.

29. $2[3x-4+x]+5=2(4x-4)+5=8x-8+5=8x-3$.

30. $6x^2$ and $3x$ are unlike terms because their variable powers differ, so they cannot be combined. The simplified expression remains $6x^2+3x$.

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F. Translating and Modeling

Objective: Build simplified algebraic expressions for verbal and financial situations.

31. $2(x+4)+5=2x+8+5=2x+13$.

32. Length= $2w+3$. P= $2(2w+3)+2w=6w+6$ feet.

33. Cost is $C=12+3m$. For $m=7$, $C=12+21=\$33$.

34. $C(d)=4.50+2.25d$. $C(8)=4.50+18=\$22.50$.

35. The integers are $n, n+1, n+2$. Their sum is $n+(n+1)+(n+2)=3n+3$.

36. After a 20% discount, the price is $0.80p$. Adding 8% tax multiplies by 1.08: $1.08(0.80p)=0.864p$.

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G. Formulas and Substitution

Objective: Use standard formulas with correct units and numerical precision.

37. $P = 2(8.5) + 2(3.2) = 17 + 6.4 = \mathbf{23.4 \text{ cm.}}$

38. $A = (1/2)(14)(9) = 7(9) = \mathbf{63 \text{ m}^2}.$

39. $d = 55(2.5) = \mathbf{137.5 \text{ miles.}}$

40. $F = (9/5)(-10) + 32 = -18 + 32 = \mathbf{14 \text{ degrees Fahrenheit.}}$

41. $V = (2.5)(4)(6.2) = 10(6.2) = \mathbf{62 \text{ ft}^3}.$

42. $I = 1500(0.04)(3) = 60(3) = \mathbf{\$180}.$

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H. Units, Rates, and Proportional Reasoning

Objective: Use conversion factors and unit rates to solve quantitative problems.

43. $7.5 \text{ ft} \cdot (12 \text{ in}/1 \text{ ft}) = \mathbf{90 \text{ inches}}$; feet cancel.

44. $90 \text{ km/hr} \cdot (1 \text{ hr}/60 \text{ min}) = 90/60 = \mathbf{1.5 \text{ km/min}}$.

45. $156 \text{ miles} \div 6 \text{ gallons} = \mathbf{26 \text{ miles per gallon}}$.

46. $\text{Rate} = 2.5/20 = 0.125 \text{ cup per cookie}$. For 32 cookies:
 $0.125(32) = \mathbf{4 \text{ cups}}$.

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I. Algebraic Reasoning and Challenge

Objective: Analyze errors, prove a pattern, determine a parameter, and model a geometric expression.

47. The likely error is adding $2+3$ before multiplying or mishandling the exponent. Correct order: $(4-1)=3$, $3^2=9$, $3(9)=27$, and $2+27=29$.

48. Let the integers be n , $n+1$, $n+2$. Their sum is $3n+3=3(n+1)$. Since $n+1$ is an integer, the sum is 3 times an integer and is therefore divisible by 3.

49. $3(kx-4)+2x=(3k+2)x-12$. Match the x -coefficient with 20: $3k+2=20$, so $3k=18$ and $k=6$. Equal coefficients make the expressions identical for every x .

50. Outer area $=(x+6)(x+4)=x^2+10x+24$. Opening area $=x(x-2)=x^2-2x$. Frame area $=12x+24$ **square units**. Physical dimensions require $x>2$.