

Chapter 2 - Exploring the Realm of Real Numbers

A. Classifying Real Numbers

Objective: Classify values within nested real-number subsets and justify irrationality.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Number sets natural $\subset$ whole $\subset$ integer $\subset$ rational $\subset$ real irrational = real but not rational	Exponent rules $a^m a^n = a^{m+n}$; $a^m / a^n = a^{m-n}$ $a^{-n} = 1/a^n$
1. Classify 0 in every applicable set: natural, whole, integer, rational, irrational, and real. Use the convention natural = {1, 2, 3, ...}.	2. Classify -13 in every applicable real-number subset.
3. Classify 7/9 in every applicable real-number subset.	4. Classify $\sqrt{121}$ in every applicable real-number subset.
5. Classify $\sqrt{18}$ as rational or irrational and explain why.	6. Draw or describe a nesting diagram showing the relationships among natural, whole, integer, rational, and real numbers.

Chapter 2 - Exploring the Realm of Real Numbers

B. Decimal Forms and Rationality

Objective: Convert terminating and repeating decimals and distinguish rational from irrational behavior.

7. Write 0.375 as a fraction in simplest form.

8. Write $2.1(6)$, where only the 6 repeats, as a fraction in simplest form.

9. Determine whether $0.101001000100001\dots$ is rational or irrational. Justify from its decimal behavior.

10. Give an example of an irrational number between 4 and 5 and prove it lies in that interval.

11. Without a calculator, decide whether $\sqrt{70}$ lies closer to 8 or 9. Explain.

12. A student says every nonterminating decimal is irrational. Give a counterexample and correct the statement.

Chapter 2 - Exploring the Realm of Real Numbers

C. Ordering and Locating Real Numbers

Objective: Compare exact values and place irrational numbers on the number line.

13. Order from least to greatest: $-\sqrt{10}$, -3.1 , $-\pi$, and $-\frac{22}{7}$.

14. Place $\sqrt{45}$ between two consecutive integers.

15. Compare using $<$, $>$, or $=$: $\frac{5}{8}$ ___ 0.62 .

16. Order from least to greatest: 1.4 , $\sqrt{2}$, $\frac{7}{5}$, and 1.41 .

17. Find the distance between -7 and 5 on a number line.

18. A point is 6 units from -2 . List all possible coordinates.

Chapter 2 - Exploring the Realm of Real Numbers

D. Absolute Value and Distance

Objective: Interpret absolute value as distance and reason about signed quantities.

19. Evaluate $|3-11|$.

20. Simplify $-|5-12|+4$.

21. Evaluate $|a-b|$ when $a=-3.5$ and $b=2.1$, and interpret the result as distance.

22. The temperature changes from -8 degrees to 7 degrees. Find the total increase and express it using absolute value.

23. For $x < 0$, simplify $|x|$ and explain your answer.

24. Error analysis: A student claims $|-9|=-9$ because the number inside is negative. Correct the reasoning.

Chapter 2 - Exploring the Realm of Real Numbers

E. Operations with Real Numbers

Objective: Perform accurate operations with integers, fractions, and decimals.

25. Compute $-18+27-14$.

26. Compute $(-6)(-9)+(-8)$.

27. Compute $72\div(-9)-5$.

28. Compute $3/4-5/6$ and write the answer in simplest form.

29. Compute $(-7/10)(15/14)$.

30. Compute $2.5-(-3.75)+1.2$.

Chapter 2 - Exploring the Realm of Real Numbers

F. Properties and Inverses

Objective: Use operational properties, identities, inverses, and closure.

31. Use properties, not a calculator, to compute $47 + (-19) + 53$ efficiently.

32. Use the distributive property to compute $18(99)$ mentally.

33. State the additive inverse and multiplicative inverse of $-5/7$.

34. Is division closed in the set of integers? Give a counterexample and explain.

35. Simplify $6x + 0$ and name the property illustrated.

36. Simplify $(3/8)(8/3)y$ and name the properties that justify the result.

Chapter 2 - Exploring the Realm of Real Numbers

G. Integer Exponents

Objective: Apply product, quotient, power, zero, and negative-exponent rules.

37. Simplify $2^5 \cdot 2^3$.

38. Simplify x^9/x^4 for x not equal to 0.

39. Simplify $(3a^2b^3)^2$.

40. Rewrite 5^{-3} with a positive exponent and evaluate.

41. Simplify $(2x^{-2}y^3)/(4y^{-1})$ using positive exponents.

42. Evaluate and compare $(-2)^4$ and -2^4 . Explain the effect of parentheses.

Chapter 2 - Exploring the Realm of Real Numbers

H. Scientific Notation

Objective: Convert and compute with quantities written in scientific notation.

43. Write 0.0000725 in scientific notation.

44. Write 6.03×10^5 in standard notation.

45. Compute $(3 \times 10^4)(2.5 \times 10^{-3})$ in scientific notation.

46. The mass of a cell is 4.2×10^{-9} gram. Find the mass of 3×10^6 such cells in scientific notation.

Chapter 2 - Exploring the Realm of Real Numbers

I. Real-Number Reasoning and Challenge

Objective: Analyze invalid rules, prove irrationality, bound square roots, and compare geometric measures.

47. Error analysis: A student writes $\sqrt{36+64}=\sqrt{36}+\sqrt{64}=14$. Find the correct value and explain why the rule used is invalid.

48. Prove that the product of a nonzero rational number and an irrational number is irrational.

49. Find all integers n for which $\sqrt{n}$ lies strictly between 5 and 6. Explain your boundary reasoning.

50. A square has area 50 square units. A second square has rational side length and area greater than 50 but as small as possible among integer areas. Find both exact side lengths and compare their perimeters.