Name: ______________________

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Answer Key: Chapter 2 - Exploring the Realm of Real Numbers

A. Classifying Real Numbers

Objective: Classify values within nested real-number subsets and justify irrationality.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Number sets natural ⊂ whole ⊂ integer ⊂ rational ⊂ real irrational = real but not rational	**Exponent rules** $a^m a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$ $a^{-n} = 1/a^n$

1. With natural={1,2,...}, 0 is **whole, integer, rational, and real**. It is rational because 0=0/1. It is not natural under the stated convention and is not irrational.	**2.** -13 is an **integer, rational, and real** number. It is not natural or whole, and it is not irrational.
3. 7/9 is **rational and real**. It is not an integer, whole, natural, or irrational number.	**4.** sqrt(121)=11, so it is **natural, whole, integer, rational, and real**.
5. 18 is not a perfect square. sqrt(18)=3sqrt(2), and sqrt(2) is irrational; therefore **sqrt(18) is irrational**.	**6.** The nesting is **natural ⊂ whole ⊂ integers ⊂ rational ⊂ real**. Irrational numbers are also real but lie outside the rational set.

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B. Decimal Forms and Rationality

Objective: Convert terminating and repeating decimals and distinguish rational from irrational behavior.

7. 0.375=375/1000. Divide by 125: **3/8**.	**8.** Let x=2.1666.... Then 10x=21.666... and 100x=216.666.... Subtract: 90x=195, so x=195/90=**13/6**.
9. The decimal is nonterminating and not eventually repeating because the zero blocks continually grow. Therefore it is **irrational**.	**10.** Example: **sqrt(17)**. Since 16<17<25, taking positive square roots gives 4<sqrt(17)<5.
11. 8^2=64 and 9^2=81. Compare 70 with the midpoint in squared distance: 70-64=6 and 81-70=11, so sqrt(70) is **closer to 8**.	**12.** Counterexample: 0.333...=1/3 is nonterminating but rational. Correct statement: a decimal is rational exactly when it **terminates or eventually repeats**.

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C. Ordering and Locating Real Numbers

Objective: Compare exact values and place irrational numbers on the number line.

13. Approximations: -sqrt(10)≈-3.1623, -22/7≈-3.142857, -pi≈-3.141593, -3.1. Thus **-sqrt(10)<-22/7<-pi<-3.1**.	**14.** 36<45<49, so **6<sqrt(45)<7**.
15. 5/8=0.625, so **5/8>0.62**.	**16.** 7/5=1.4. Since 1.41<sqrt(2)≈1.4142, the order is **1.4=7/5<1.41<sqrt(2)**.
17. Distance=|5-(-7)|=|12|=**12 units**.	**18.** Coordinates x satisfy |x-(-2)|=6, so x=-2±6. Thus **x=-8 or x=4**.

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D. Absolute Value and Distance

Objective: Interpret absolute value as distance and reason about signed quantities.

19. $\lvert 3-11\rvert=\lvert -8\rvert=$**8**.	**20.** $-\lvert 5-12\rvert+4=-\lvert -7\rvert+4=-7+4=$**-3**.
21. $\lvert a-b\rvert=\lvert -3.5-2.1\rvert=\lvert -5.6\rvert=$**5.6 units**, the separation between the two points.	**22.** Increase=7-(-8)=15. As a distance, **$\lvert 7-(-8)\rvert=15$ degrees**.
23. If $x<0$, its opposite $-x$ is positive. Therefore **$\lvert x\rvert=-x$**.	**24.** Absolute value is distance from 0 and cannot be negative. Thus **$\lvert -9\rvert=9$**.

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E. Operations with Real Numbers

Objective: Perform accurate operations with integers, fractions, and decimals.

25. -18+27-14=9-14=**-5**.	**26.** (-6)(-9)+(-8)=54-8=**46**.
27. 72÷(-9)-5=-8-5=**-13**.	**28.** 3/4-5/6=9/12-10/12=**-1/12**.
29. (-7/10)(15/14). Cancel 7 with 14 and 15 with 10: -(1·3)/(2·2)=**-3/4**.	**30.** 2.5-(-3.75)+1.2=2.5+3.75+1.2=**7.45**.

Answer Key: Chapter 2 - Exploring the Realm of Real Numbers

F. Properties and Inverses

Objective: Use operational properties, identities, inverses, and closure.

31. Regroup 47+53+(-19)=100-19=**81**, using commutative and associative properties.	**32.** 18(99)=18(100-1)=1800-18=**1782**.
33. The additive inverse is **5/7**. The multiplicative inverse is the reciprocal **-7/5**.	**34.** No. Example: 1 and 2 are integers, but 1÷2=1/2 is not an integer. Thus integers are **not closed under division**.
35. 6x+0=**6x** by the additive identity property.	**36.** (3/8)(8/3)y=1·y=**y**, using multiplicative inverses, associativity, and the multiplicative identity.

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G. Integer Exponents

Objective: Apply product, quotient, power, zero, and negative-exponent rules.

37. Same base: add exponents. $2^5 \cdot 2^3 = 2^8 = \mathbf{256}$.	**38.** Subtract exponents: $x^9/x^4 = \mathbf{x^5}$, x not equal to 0.
39. Apply the power to every factor: $3^2a^4b^6 = \mathbf{9a^4b^6}$.	**40.** $5^{-3} = 1/5^3 = \mathbf{1/125}$.
41. $(2/4)x^{-2}y^4 = (1/2)(y^4/x^2) = \mathbf{y^4/(2x^2)}$.	**42.** $\mathbf{(-2)^4 = 16}$, while $\mathbf{-2^4 = -(16) = -16}$. Parentheses make -2 the base; without them, the exponent applies only to 2 and the leading negative remains outside.

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H. Scientific Notation

Objective: Convert and compute with quantities written in scientific notation.

43. Move the decimal 5 places right: $\mathbf{7.25\times10^{-5}}$.	**44.** Move the decimal 5 places right: **603,000**.
45. Multiply coefficients and add exponents: $3(2.5)\times10^{1}=7.5\times10=\mathbf{7.5\times10^{1}}$.	**46.** $(4.2\times10^{-9})(3\times10^{6})=12.6\times10^{-3}=\mathbf{1.26\times10^{-2}}$ **gram**.

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I. Real-Number Reasoning and Challenge

Objective: Analyze invalid rules, prove irrationality, bound square roots, and compare geometric measures.

47. sqrt(36+64)=sqrt(100)=**10**. Square roots do not distribute over addition; indeed, 10 is not 6+8.

48. Let r be a nonzero rational and u irrational. Suppose ru is rational, say q. Then u=q/r. Rational numbers are closed under division by a nonzero rational, so u would be rational, a contradiction. Hence **ru is irrational**.

49. 5<sqrt(n)<6. Squaring positive quantities gives 25<n<36. The integers are **26,27,28,29,30,31,32,33,34,35**.

50. First square side=**sqrt(50)=5sqrt(2)**, perimeter=20sqrt(2). The least greater integer area that is a perfect square is 64? No: integer area 51 has irrational side. A rational side squared to an integer must give a perfect-square integer, so the next is **64**, side 8, perimeter 32. Since 20sqrt(2)≈28.28, the second perimeter is larger by **32-20sqrt(2)**.