

## Answer Key: Chapter 3 - Navigating Linear Equations

### A. One-Step Equations

**Objective:** Use inverse operations and check solutions.

**Directions:** Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

| <b>Equality principle</b><br>Perform the same operation on both sides.<br>Check by substitution in the original equation. | <b>Solution types</b><br>true statement → infinitely many<br>false statement → no solution<br>$x = \text{value}$ → one solution |
|---------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <b>1.</b> $x + 9 = 23 \rightarrow x = 14$ . Check: $14 + 9 = 23$ . <b><math>x = 14</math>.</b>                            | <b>2.</b> $y - 14 = -6 \rightarrow y = 8$ . <b><math>y = 8</math>.</b>                                                          |
| <b>3.</b> $-7m = 42 \rightarrow m = -6$ .                                                                                 | <b>4.</b> $n/8 = -5 \rightarrow n = -40$ .                                                                                      |
| <b>5.</b> $p = 7.3 + 2.5 = 9.8$ .                                                                                         | <b>6.</b> Equation $n + 5 = 19$ , so <b><math>n = 14</math></b> .                                                               |

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### B. Two-Step Equations

**Objective:** Solve equations with signed, fractional, and decimal coefficients.

7.  $4x=28 \rightarrow x=7$ .

8.  $-3y=21 \rightarrow y=-7$ .

9.  $z/5=-4 \rightarrow z=-20$ .

10.  $-2.4a=7.2 \rightarrow a=-3$ .

11.  $(3/4)x=12 \rightarrow x=12(4/3)=16$ .

12. Undo addition first:  $6x=24$ , then divide by 6.  **$x=4$** . Dividing first is possible only if every term is divided by 6 correctly.

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### C. Distribution and Grouping

**Objective:** Solve equations requiring distribution, grouping, and denominator clearing.

**13.**  $x+5=9 \rightarrow x=4$ .

**14.**  $-8y+12=28 \rightarrow -8y=16 \rightarrow y=-2$ .

**15.**  $10m+5-3=42 \rightarrow 10m=40 \rightarrow m=4$ .

**16.**  $2(3x-x+4)=24 \rightarrow 4x+8=24 \rightarrow x=4$ .

**17.**  $7-6p-10=-21 \rightarrow -6p-3=-21 \rightarrow -6p=-18 \rightarrow p=3$ .

**18.** Multiply by 6:  $2(x-2)+3(x+1)=42$ . Then  $5x-1=42$ , so  $x=43/5$ .

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### D. Variables on Both Sides

**Objective:** Collect variable terms strategically and solve accurately.

19.  $3x=21 \rightarrow x=7$ .

20.  $-7y=21 \rightarrow y=-3$ .

21.  $12x-6=4x+18 \rightarrow 8x=24 \rightarrow x=3$ .

22.  $6a+15-4=5a-10+18 \rightarrow 6a+11=5a+8 \rightarrow a=-3$ .

23.  $9=0.3x \rightarrow x=30$ .

24. Subtract  $(1/6)x$ :  $(1/2)x+5=-1 \rightarrow (1/2)x=-6 \rightarrow x=-12$ .

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### E. Identities and Contradictions

**Objective:** Distinguish one solution, no solution, and infinitely many solutions.

**25.** Both sides simplify to  $4x+8$ , so the equation is true for every real  $x$ : **infinitely many solutions.**

**26.**  $6x-15=6x+1$  gives  $-15=1$ , false. Therefore **no solution.**

**27.** Right side is  $5x-10+3=5x-7$ , identical to the left. Therefore **infinitely many solutions.**

**28.** For identity, coefficients and constants must match.  $kx-4k=3x-12$  gives  **$k=3$ .**

**29.**  $2x+6=2x+c$  reduces to  $6=c$ . For no solution, **any  $c$  not equal to 6.**

**30.** Subtracting  $7x$  gives  $5=5$ , which is always true and imposes no restriction. The solution set is **all real numbers**, not  $x=0$ .

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### F. Proportions and Scale Models

**Objective:** Solve proportions and use constant ratios in applications.

**31.**  $18x=84 \rightarrow x=84/18=\mathbf{14/3}$ .

**32.** Restriction  $y \neq 1$ . Cross multiply:  $15=2(y-1)=2y-2$ , so  $2y=17$  and  $y=\mathbf{17/2}$ .

**33.**  $9(x+4)=6(2x-1) \rightarrow 9x+36=12x-6 \rightarrow 42=3x \rightarrow x=\mathbf{14}$ .

**34.**  $1/24=3.75/d \rightarrow d=24(3.75)=\mathbf{90 \text{ miles}}$ .

**35.**  $6/8=h/30 \rightarrow 8h=180 \rightarrow h=\mathbf{22.5 \text{ feet}}$ .

**36.**  $3.5/14=x/22 \rightarrow x=22(3.5/14)=22(1/4)=\mathbf{5.5 \text{ cups}}$ .

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### G. Literal Equations

**Objective:** Rearrange formulas while tracking restrictions.

**37.**  $P - 2w = 2l$ , so  $l = (P - 2w)/2 = P/2 - w$ .

**38.**  $2A = bh$ , so  $h = 2A/b$ ,  $b \neq 0$ .

**39.**  $t = d/r$ . With  $d = 315$  and  $r = 70$ ,  $t = 315/70 = 4.5$  hours.

**40.**  $F - 32 = (9/5)C$ , so  $C = (5/9)(F - 32)$ .

**41.** Divide by  $\pi r^2$ :  $h = V/(\pi r^2)$ ,  $r \neq 0$ .

**42.**  $ax + b = cx + d \rightarrow (a - c)x = d - b$ , so  $x = (d - b)/(a - c)$ .

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### H. Linear Equation Modeling

**Objective:** Define a variable, build an equation, and interpret the solution.

**43.** Let integers be  $n, n+1, n+2$ . Then  $3n+3=72 \rightarrow n=23$ . Integers: **23, 24, 25**.

**44.**  $35+18m=197 \rightarrow 18m=162 \rightarrow$  **9 months**.

**45.** Let width  $w$ , length  $2w+5$ . Then  $2w+2(2w+5)=58 \rightarrow 6w=48 \rightarrow$  width **8**, length **21**.

**46.**  $42+0.08t=49.20 \rightarrow 0.08t=7.20 \rightarrow$  **90 extra texts**.

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### I. Reasoning, Proof, and Challenge

**Objective:** Analyze errors, justify equality steps, determine parameters, and compare linear models.

**47.**  $4-6x+15=31 \rightarrow -6x+19=31 \rightarrow -6x=12 \rightarrow \mathbf{x=-2}$ . A common error is writing  $-3(2x-5)=-6x-15$ ; the second product is  $+15$ .

**48.** From  $5x-2=5y-2$ , add 2 to both sides (addition property):  $5x=5y$ . Divide both sides by 5 (division property):  $\mathbf{x=y}$ .

**49.** Left side  $=(2k-4)x+6$ . For identity with  $10x+6$ ,  $2k-4=10$ , so  $\mathbf{k=7}$ . For every  $k \neq 7$ , unequal x-coefficients give the single solution  $x=0$ .

**50.**  $420-18t=150+9t \rightarrow 270=27t \rightarrow \mathbf{t=10 \text{ minutes}}$ . Common amount:  $420-180=240$ ;  $150+90=240$ , so **240 liters**.