

Chapter 4 - Visualizing Relations and Functions

A. Relations and Function Rules

Objective: Identify domain, range, inputs, outputs, and the defining function criterion.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Function criterion Each input x has exactly one output y . Graphs must pass the vertical-line test.	Key language domain = inputs; range = outputs $f(a)$ means the output when $x=a$
1. For $R=\{(-3,4),(0,1),(2,4),(5,-2)\}$, state the domain and range.	2. Is $\{(1,3),(2,5),(3,5),(4,7)\}$ a function? Justify from the inputs.
3. Is $\{(-2,1),(-2,4),(0,3)\}$ a function? Explain.	4. A mapping sends $-1 \rightarrow 2$, $0 \rightarrow 2$, $1 \rightarrow 4$, and $2 \rightarrow 6$. Is it a function? State its range.
5. In the relation between hours studied and test score, identify a reasonable independent variable and dependent variable.	6. Describe one advantage of representing a relation as (a) a table and (b) a graph.

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B. Function Notation

Objective: Evaluate, solve, and interpret functions using notation and tables.

7. Let $f(x)=3x-8$. Find $f(5)$.

8. Let $g(t)=t^2-4t+1$. Find $g(-2)$.

9. For $h(x)=7-2x$, solve $h(x)=-5$.

10. If $p(x)=|x+1|$, find $p(-4)+p(2)$.

11. A function table contains $(x,f(x))=(-2,7),(0,3),(3,-6),(5,-10)$. Find $f(3)$, solve $f(x)=3$, and state the ordered pair containing $x=5$.

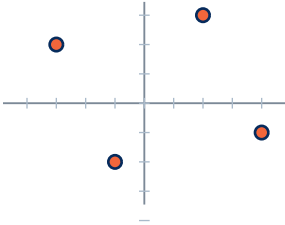
12. The function $C(n)=12+4.5n$ gives total cost in dollars. Interpret $C(6)=39$ in context.

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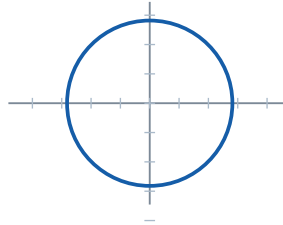
C. Graphs and the Vertical-Line Test

Objective: Read plotted relations and determine whether each graph represents a function.

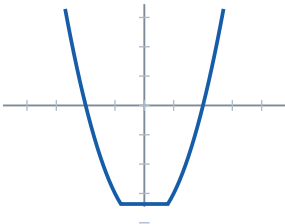
- 13.** Use the plotted relation. List its domain and range, then decide whether it is a function.



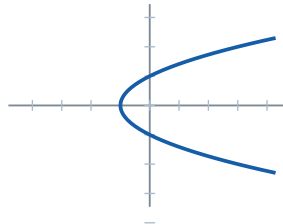
- 14.** Use the graph. Does the relation define y as a function of x ? Apply the vertical-line test.



- 15.** Use the graph of $y = x^2 - 4$. State the y -intercept and whether the graph represents a function.



- 16.** Use the sideways parabola. Explain why it fails the vertical-line test.



- 17.** A graph includes closed points $(-3, 2)$ and $(4, -1)$ and every point on the line segment between them. State the domain and range in interval notation.

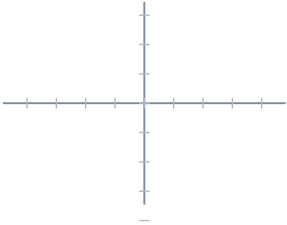
- 18.** A relation is given by $x = y^2$. Explain why y is not a function of x , then state two y -values paired with $x = 9$.

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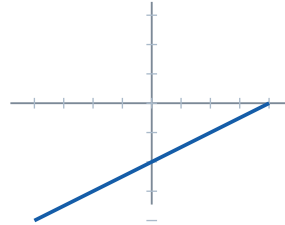
D. Building and Reading Graphs

Objective: Create tables and graphs, find intercepts, and interpret association.

- 19.** Create a table for $y = -2x + 3$ using $x = -2, -1, 0, 1, 2$, then plot the five points on the provided grid.



- 20.** Graph $y = (1/2)x - 2$ using its y-intercept and one additional point. State both points.



- 21.** Find the x- and y-intercepts of $3x + 2y = 12$.

- 22.** Graph $x = -3$ conceptually. Is $x = -3$ a function of x in the form $y = f(x)$? Explain.

- 23.** A graph passes through $(0, -1), (1, 2), (2, 5), (3, 8)$. Find a linear rule and predict the value at $x = 10$.

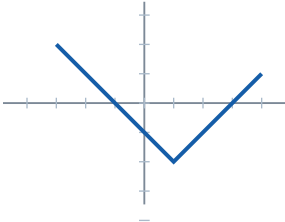
- 24.** A scatter plot shows points close to an increasing straight line. What does this indicate about the association, and does it prove causation?

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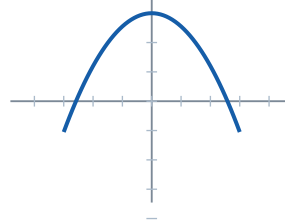
E. Features and Context of Graphs

Objective: Describe vertices, intervals, step boundaries, rates, and practical restrictions.

25. For the V-shaped graph shown, state the vertex, domain, range, and intervals where the function decreases and increases.



26. For the parabola shown, state the vertex, axis of symmetry, domain, and range.



27. A step function assigns shipping cost \$5 for $0 < w \leq 2$, \$8 for $2 < w \leq 5$, and \$12 for $5 < w \leq 10$. Find the cost at $w = 2$, 2.1, 5, and 7 pounds.

28. A distance-time graph passes through $(0,0)$, $(2,90)$, $(5,225)$. Find the average rate on each interval and interpret the shape.

29. The height of a thrown ball is graphed only from launch until it hits the ground. Explain why the practical domain and range differ from the domain and range of the quadratic formula itself.

30. Two graphs have the same y-intercept. Graph A rises 6 units for every 2 units right; Graph B rises 5 units for every 5 units right. Which grows faster? Quantify both rates.

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F. Linear and Nonlinear Patterns

Objective: Use first differences and multiplicative patterns to distinguish function families.

31. Determine whether the table is linear: x : 0,1,2,3; y : 5,8,11,14. If linear, give the rate and rule.

32. Determine whether the table is linear: x : -2,-1,0,1; y : 4,1,0,1. Explain using first differences.

33. Find a rule for the table x : 1,2,3,4; y : 6,11,16,21.

34. The table x : 0,1,2,3 has y : 2,6,18,54. Is it linear? Describe the multiplicative pattern.

35. Compare $f(x)=4x-1$ and $g(x)=x^2$ at $x=0,1,2,3$. At which listed x -values is $g(x)>f(x)$?

36. A tank contains 200 liters and loses 15 liters per minute. Write the function, state its realistic domain, and identify whether it is linear.

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G. Function Operations and Transformations

Objective: Connect sequences, models, inverses, compositions, shifts, and piecewise rules.

37. The sequence 7,11,15,19,... is represented by a function of term number n . Write $f(n)$ and find $f(25)$.

38. A ride costs $C(m)=3.25+1.80m$. Interpret the slope and intercept, and find $C(12)$.

39. Interchange x and y in $y=2x-5$ and solve for y . What relation results?

40. Given $f(x)=2x+1$ and $g(x)=x^2$, find $f(g(3))$ and $g(f(3))$.

41. Describe how the graph of $y=|x|$ changes to produce $y=|x-3|+2$.

42. Evaluate the piecewise function $f(x)=x+4$ for $x<0$ and $f(x)=x^2$ for $x\geq 0$ at $x=-3, 0$, and 2 .

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H. Modeling Domains and Representations

Objective: Choose discrete, continuous, or piecewise models and interpret average rate.

43. A school charges \$8 per ticket for up to 100 tickets, then \$6 per ticket for each ticket beyond 100. Explain why total revenue is naturally modeled piecewise.

44. A relation models the number of books purchased and total cost. Should its graph be discrete or continuous? Explain.

45. For $A(r) = \pi r^2$, state a practical domain when r is a circle's radius and explain the restriction.

46. For $f(x) = x^2 - 1$, find the average rate of change from $x=1$ to $x=4$ and interpret it as a secant slope.

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I. Function Reasoning and Challenge

Objective: Correct misconceptions, prove the graph test, analyze parameters, and model a step function.

47. Error analysis: A student says a relation is a function because no y-value repeats. Correct the criterion and give a counterexample.

48. Prove that every nonvertical line in the coordinate plane represents a function of x , while a vertical line does not.

49. Find all values of k for which $R = \{(1,2), (k,5), (3,7)\}$ is a function. Explain the excluded value(s).

50. A parking garage charges \$5 for the first hour and \$3 for each additional hour or part of an hour, up to 6 hours. Define a suitable step-function model, state its domain, and find the charges for 1, 1.2, 3, and 5.5 hours.