

Answer Key: Chapter 4 - Visualizing Relations and Functions

A. Relations and Function Rules

Objective: Identify domain, range, inputs, outputs, and the defining function criterion.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Function criterion Each input x has exactly one output y . Graphs must pass the vertical-line test.	Key language domain = inputs; range = outputs $f(a)$ means the output when $x=a$
1. Domain=$\{-3,0,2,5\}$. Range=$\{-2,1,4\}$; repeated outputs are listed once.	2. Yes. Each input appears exactly once and is paired with one output.
3. No. Input -2 is paired with both 1 and 4.	4. Yes; every input has one arrow. Range=$\{2,4,6\}$.
5. Independent: hours studied. Dependent: test score, because score is viewed as responding to study time.	6. A table displays exact input-output values compactly. A graph makes overall shape, trend, intercepts, and gaps visually apparent.

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B. Function Notation

Objective: Evaluate, solve, and interpret functions using notation and tables.

7. $f(5)=3(5)-8=7$.

8. $g(-2)=(-2)^2-4(-2)+1=4+8+1=13$.

9. $7-2x=-5 \rightarrow -2x=-12 \rightarrow x=6$.

10. $p(-4)=|-3|=3$ and $p(2)=|3|=3$; sum=**6**.

11. **$f(3)=-6$** ; $f(x)=3$ when **$x=0$** ; ordered pair with $x=5$ is **$(5,-10)$** .

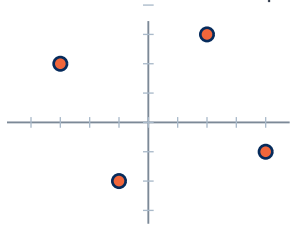
12. Six units/items produce a total cost of **\$39**, including the fixed \$12 and 6 variable charges of \$4.50.

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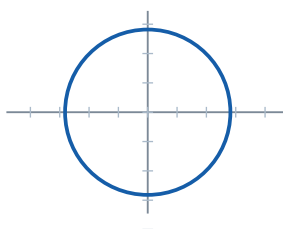
C. Graphs and the Vertical-Line Test

Objective: Read plotted relations and determine whether each graph represents a function.

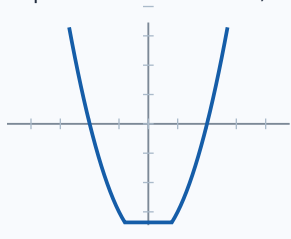
13. Plotted points are $(-3,2), (-1,-2), (2,3), (4,-1)$.
Domain = $\{-3, -1, 2, 4\}$; range = $\{-2, -1, 2, 3\}$. It is a function
because x-values do not repeat.



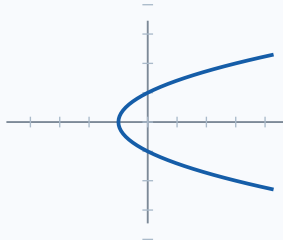
14. No. Many vertical lines through the circle meet it twice, so one input has two outputs.



15. At $x=0$, $y=-4$, so y-intercept = $(0, -4)$. Every vertical line meets the parabola at most once, so it is a function.



16. For many x-values to the right of the vertex, a vertical line intersects the sideways parabola twice; therefore one x has two y-values and the relation is **not a function**.



17. Closed segment endpoints give domain $[-3, 4]$. The y-values run from -1 to 2, so range $[-1, 2]$.

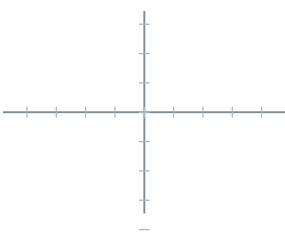
18. For $x > 0$, $y = \pm\sqrt{x}$, giving two outputs. At $x=9$, $y=3$ and $y=-3$.

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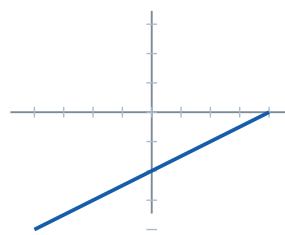
D. Building and Reading Graphs

Objective: Create tables and graphs, find intercepts, and interpret association.

19. Values: $(-2,7), (-1,5), (0,3), (1,1), (2,-1)$. Plotting them produces a straight line.



20. Y-intercept **$(0,-2)$** . With slope $1/2$, move right 2 and up 1 to **$(2,-1)$** .



21. Set $y=0$: $3x=12 \rightarrow x=4$, so x-intercept **$(4,0)$** . Set $x=0$: $2y=12 \rightarrow y=6$, so y-intercept **$(0,6)$** .

22. It is a vertical line through all points $(-3,y)$. It is **not** $y=f(x)$, because $x=-3$ is paired with infinitely many y -values.

23. Outputs increase by 3 per unit x and $f(0)=-1$, so **$f(x)=3x-1$** . $f(10)=29$.

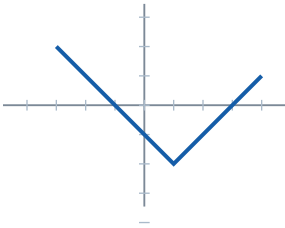
24. The plot indicates a **strong positive linear association**. Association alone does not prove that one variable causes the other.

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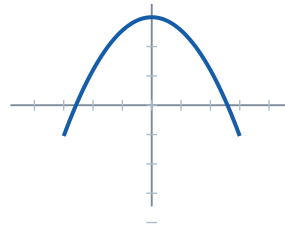
E. Features and Context of Graphs

Objective: Describe vertices, intervals, step boundaries, rates, and practical restrictions.

25. Vertex=**(1,-2)**; domain all real numbers; range **[-2,infinity)**. Decreasing on $(-\infty,1)$ and increasing on $(1,\infty)$.



26. Vertex=**(0,3)**; axis $x=0$; domain all real numbers; range **$(-\infty,3]$** because it opens downward.



27. At $w=2$: **\$5**; 2.1: **\$8**; 5: **\$8**; 7: **\$12**. Endpoint symbols determine the boundary values.

28. 0 to 2: $90/2=45$ **units/hour**. 2 to 5: $(225-90)/(5-2)=135/3=45$ **units/hour**. Equal rates mean a straight line and constant speed.

29. The quadratic formula may accept every real time input and extend below ground, but the physical model uses only times from launch to impact. Its range uses only heights attained during that interval.

30. A rate= $6/2=3$. B rate= $5/5=1$. Graph A grows faster.

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F. Linear and Nonlinear Patterns

Objective: Use first differences and multiplicative patterns to distinguish function families.

31. First differences are +3, so linear. Rate=**3**; y-intercept 5; rule **$y=3x+5$** .

32. First differences are -3,-1,+1, not constant, so the table is **not linear**.

33. Difference is 5, so $y=5x+b$. Using (1,6), $b=1$. Rule: **$y=5x+1$** .

34. Not linear; outputs multiply by **3** for each increase of 1 in x . It shows exponential growth.

35. Values f : -1,3,7,11; g : 0,1,4,9. Thus $g>f$ only at **$x=0$** among the listed values.

36. **$V(t)=200-15t$** . Realistically $0 \leq t \leq 200/15=40/3$ minutes. It is linear with rate -15 L/min.

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G. Function Operations and Transformations

Objective: Connect sequences, models, inverses, compositions, shifts, and piecewise rules.

37. Arithmetic sequence: $f(n)=7+4(n-1)=4n+3$. $f(25)=103$.

38. Slope 1.80 is the cost per mile; intercept 3.25 is the initial charge. $C(12)=3.25+21.60=\$24.85$.

39. $x=2y-5 \rightarrow x+5=2y \rightarrow y=(x+5)/2$, the inverse relation.

40. $g(3)=9$, so $f(g(3))=f(9)=19$. $f(3)=7$, so $g(f(3))=g(7)=49$.

41. Shift $y=|x|$ **right 3 units** and **up 2 units**; the new vertex is (3,2).

42. $f(-3)=-3+4=1$; $f(0)=0^2=0$; $f(2)=2^2=4$.

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H. Modeling Domains and Representations

Objective: Choose discrete, continuous, or piecewise models and interpret average rate.

43. The unit price changes after 100 tickets, so one rule applies for $n \leq 100$ and another for $n > 100$. A continuous total would be $R(n) = 8n$ for $n \leq 100$ and $R(n) = 800 + 6(n - 100)$ for $n > 100$, with integer n .

44. Discrete. Books are counted in whole numbers; fractional book purchases are not part of the model.

45. Practical domain $r \geq 0$ (often $r > 0$ for a nondegenerate circle). Negative radii have no geometric meaning.

46. $[f(4) - f(1)] / (4 - 1) = [15 - 0] / 3 = 5$, the slope of the secant through (1,0) and (4,15).

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I. Function Reasoning and Challenge

Objective: Correct misconceptions, prove the graph test, analyze parameters, and model a step function.

47. A function requires that **no input x repeat with different outputs**; outputs may repeat. Counterexample function: $\{(1,5),(2,5)\}$.

48. A nonvertical line can be written $y=mx+b$, which assigns exactly one y to every x , so it passes the vertical-line test. A vertical line $x=c$ contains all points (c,y) , so input c has many outputs and fails.

49. The only possible repeated input is $k=1$ or $k=3$. If $k=1$, input 1 has outputs 2 and 5; if $k=3$, input 3 has outputs 5 and 7. Therefore **$k \neq 1$ and $k \neq 3$** .

50. Let h be hours with $0 < h \leq 6$. Charge **$C(h)=5+3(\text{ceil}(h)-1)$** , where $\text{ceil}(h)$ is the least integer $\geq h$. Thus $C(1)=\$5$, $C(1.2)=\$8$, $C(3)=\$11$, and $C(5.5)=\$20$.