

Answer Key: Chapter 5 - Unraveling Linear Equations

A. Slope as Rate of Change

Objective: Calculate and interpret positive, negative, zero, and undefined slopes.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Slope $m = (y_2 - y_1) / (x_2 - x_1)$ positive: rises; negative: falls	Equation forms $y = mx + b$ $y - y_1 = m(x - x_1)$ $Ax + By = C$
1. $m = (13 - 3) / (7 - 2) = 10 / 5 = 2$.	2. $m = (-1 - 5) / (2 - (-4)) = -6 / 6 = -1$.
3. $m = (6 - 6) / (8 - (-3)) = 0$.	4. Denominator $4 - 4 = 0$, so slope is undefined .
5. $m = \text{rise/run} = -12 / 5 = -12/5$; the line decreases left to right.	6. $(9 - 1) / (-2 - 6) = 8 / -8 = -1$, or $(1 - 9) / (6 - (-2)) = -8 / 8 = -1$. Both give -1 because numerator and denominator signs reverse together.

Answer Key: Chapter 5 - Unraveling Linear Equations

B. Slope-Intercept Form

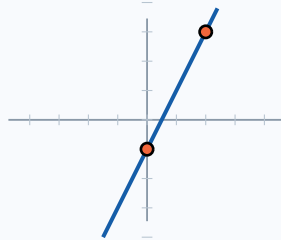
Objective: Identify, write, and graph equations in $y=mx+b$ form.

7. Slope **4**; y-intercept **(0,-7)**.

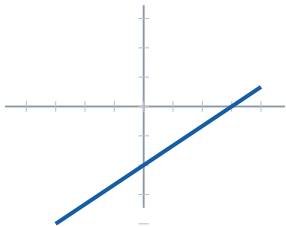
8. $2y=-3x+10 \rightarrow y=(-3/2)x+5$. Slope $-3/2$; intercept $(0,5)$.

9. $y=-3x+5$.

10. The graph passes through $(0,-1)$ and $(2,3)$. Slope $=(3+1)/2=2$, so $y=2x-1$.



11. Start $(0,-2)$; move right 3/up 2: $(3,0)$; again: $(6,2)$. Three points: **(0,-2),(3,0),(6,2)**.



12. $y=0x+6$; slope 0 and y-intercept $(0,6)$. Its graph is horizontal.

Answer Key: Chapter 5 - Unraveling Linear Equations

C. Point-Slope and Two-Point Forms

Objective: Write linear equations from a slope and point or from two points.

13. $y+2=4(x-3)$.

14. $y+5=-2x+2 \rightarrow y=-2x-3$.

15. $y-7=(-1/2)(x-2) \rightarrow y=(-1/2)x+1+7=y=(-1/2)x+8$.

16. $m=(15-3)/(2-(-4))=12/6=2$. Using $(-4,3)$: $y-3=2(x+4)$, so $y=2x+11$.

17. Both points have $x=5$, so the line is $x=5$. It is vertical, has undefined slope, and cannot be written $y=mx+b$.

18. For any point (x,y) , the slope from (a,b) is $(y-b)/(x-a)=m$. Multiplying gives $y-b=m(x-a)$.

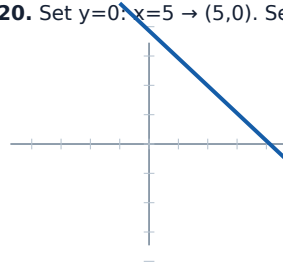
Answer Key: Chapter 5 - Unraveling Linear Equations

D. Standard Form and Intercepts

Objective: Convert forms and use axis intercepts in algebraic and contextual problems.

19. Add $3x$: **$3x+y=8$** .

20. Set $y=0$: $x=5 \rightarrow (5,0)$. Set $x=0$: $y=4 \rightarrow (0,4)$.



21. Slope $= (-3-0)/(0-6) = 1/2$, so $y = (1/2)x - 3$. Multiply by 2: **$x-2y=6$** .

22. $2y-6=5x+20 \rightarrow -5x+2y=26$. With $A>0$: **$5x-2y=-26$** .

23. Revenue: **$12x+6y=1620$** . If $y=0$, $x=135$ adult tickets; if $x=0$, $y=270$ student tickets. Intercepts are theoretical single-type sales amounts.

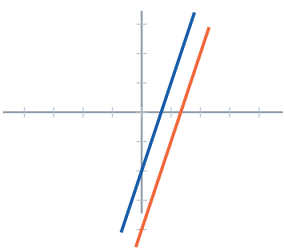
24. Intercept form is convenient when both axis intercepts are known. **$x/8+y/4=1$** , equivalently **$x+2y=8$** .

Answer Key: Chapter 5 - Unraveling Linear Equations

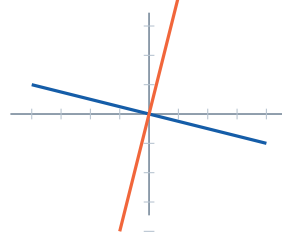
E. Parallel and Perpendicular Lines

Objective: Compare slopes and construct lines with geometric constraints.

25. Second: $-2y = -6x + 9 \rightarrow y = 3x - 9/2$. Both slopes 3 but intercepts differ, so **parallel**.



26. Slopes $-1/4$ and 4 have product -1 , so **perpendicular**.



27. $2x - 3y = 12$ has slope $2/3$. Through $(6, -1)$: $y + 1 = (2/3)(x - 6)$, so $y = (2/3)x - 5$.

28. Perpendicular slope $= -5/2$. Through $(4, 3)$: $y - 3 = (-5/2)(x - 4)$, or $y = (-5/2)x + 13$.

29. Slope $= (8 - 2)/(4 - k) = 6/(4 - k) = 3$. Then $6 = 12 - 3k \rightarrow k = 2$.

30. Let lines $y = mx + b_1$ and $y = mx + b_2$ with $b_1 \neq b_2$. An intersection would require $mx + b_1 = mx + b_2$, hence $b_1 = b_2$, a contradiction. Thus they do not intersect.

Answer Key: Chapter 5 - Unraveling Linear Equations

F. Transformations and Families of Lines

Objective: Explain how slope and intercept changes transform a linear graph.

31. Same slope, intercept decreases by 8. Shift the first graph **down 8 units**.

32. Slope is **-3**; point-slope form emphasizes point **(4,2)**. It is the line $y=-3x$ shifted to pass through that point.

33. The slope changes from 1 to -2: reflect across the x-axis and vertically stretch by factor 2; then shift **up 6**.

34. Slope m remains unchanged; b increases by 7. New equation: **$y=mx+(b+7)$** .

35. Reflection sends y to $-y$: **$y=-mx-b$** . Both slope and y -intercept change sign.

36. All equations $y=mx+b$ with fixed m have the same rate of change. Different b -values only shift the line vertically, so the lines are parallel.

Answer Key: Chapter 5 - Unraveling Linear Equations

G. Linear Modeling

Objective: Build, interpret, and compare linear models with units and practical domains.

37. $C(h)=14+6.50h$. Intercept \$14 is fixed cost; slope \$6.50/hour. $C(4.5)=14+29.25=\$43.25$.

38. $V(t)=8400-175t$. Empty when $t=8400/175=48$ min. Domain $0 \leq t \leq 48$.

39. $T(t)=68+2.5t$. $T(7)=68+17.5=85.5^\circ\text{F}$.

40. Intercept 61 predicts the score at zero study hours; slope 4.8 predicts 4.8 score points per additional hour. At 5 hours: **85**.

41. $3.75+2.40m=7.35+1.80m \rightarrow 0.60m=3.60 \rightarrow m=6$ miles.

42. Rate $=(130-250)/8=-15$ dollars/week. Model $B(w)=250-15w$.

Answer Key: Chapter 5 - Unraveling Linear Equations

H. Advanced Equation Construction

Objective: Recover linear rules from tables, intercepts, and function values.

43. Slopes: $(7-11)/(0+2)=-2$; $(1-7)/(3-0)=-2$; $(-7-1)/(7-3)=-2$. With y-intercept 7: **$y=-2x+7$** .

44. Point (9,0), slope $-2/3$: $y=(-2/3)(x-9)=$ **$(-2/3)x+6$** . Y-intercept **6**.

45. Slope (2,10) to (5,19)=3. Thus $(10-4)/(2-p)=3 \rightarrow 6=6-3p \rightarrow$ **$p=0$** .

46. Slope= $(-2-14)/(5-(-3))=-16/8=-2$. Using $f(5)=-2$ gives $-2=-10+b$, $b=8$. **$f(x)=-2x+8$** . Solve $6=-2x+8 \rightarrow$ **$x=1$** .

Answer Key: Chapter 5 - Unraveling Linear Equations

I. Reasoning, Proof, and Challenge

Objective: Correct slope errors, prove structural claims, analyze parameters, and critique extrapolation.

47. The student reversed subtraction order in only one ratio. Correctly $m = (-7-5)/(4-(-2)) = -12/6 = -2$.

48. Let points $(x_1, mx_1 + b), (x_2, mx_2 + b)$. Their midpoint y-coordinate is $[m(x_1 + x_2) + 2b]/2 = m[(x_1 + x_2)/2] + b$, exactly the line's value at midpoint x . Therefore the midpoint lies on the line.

49. $kx + 3y = 12$ has slope $-k/3$. The second line has slope 3. Perpendicular slopes multiply to -1 : $(-k/3)(3) = -k = -1$, so $k = 1$. Since $B = 3 \neq 0$, the first line is nonvertical for every k .

50. Rate $= (28500 - 24000)/6 = 750$ people/year. With t years since 2018: $P(t) = 24000 + 750t$. In 2030, $t = 12$, $P = 33,000$. Limitation: a constant rate may not continue because migration, births, policy, or economic conditions can change.