

Chapter 6 - Balancing Inequalities

A. Inequality Language and Graphs

Objective: Translate inequalities and represent solution sets on number lines and with interval notation.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Sign rule

Add/subtract: keep the symbol.

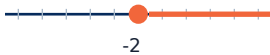
Multiply/divide by a negative: reverse it.

Compound forms

$|x| < a \rightarrow -a < x < a$

$|x| > a \rightarrow x < -a \text{ or } x > a$

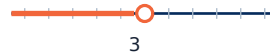
1. Write an inequality for the number-line graph shown and express it in interval notation.



2. Translate 'a number x is at least -4' into an inequality and interval notation.

3. Translate 'the temperature T is below 18 degrees' into an inequality.

4. Graph $x < 3$ on the provided number line and write interval notation.



5. List three integer solutions of $-2 \leq n < 4$.

6. Explain the difference between $x > 5$ and $x \geq 5$ in words and on a number line.

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B. One-Step Inequalities

Objective: Use inverse operations and reverse the symbol correctly for negative factors.

7. Solve $x+8>15$ and graph the solution.



8. Solve $y-12\leq-5$.

9. Solve $6m<-42$.

10. Solve $-4p\geq 20$ and explain why the inequality reverses.

11. Solve $n/(-3)<7$.

12. A student solves $-2x>10$ as $x>-5$. Correct the work and explain the rule.

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C. Multi-Step Inequalities

Objective: Distribute, combine terms, clear denominators, and solve accurately.

13. Solve $3x - 7 \leq 14$.

14. Solve $9 - 2y > 21$.

15. Solve $5(2m - 1) \geq 3m + 16$.

16. Solve $4 - 3(2x + 5) < 10$.

17. Solve $(x - 2)/4 + (x + 1)/2 \geq 5$.

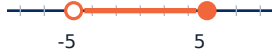
18. Solve $0.6x - 1.8 < 2.4x + 5.4$.

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D. Compound Inequalities

Objective: Solve and graph intersections (and) and unions (or).

19. Solve and graph $-3 < x + 2 \leq 7$. Write interval notation.



20. Solve and graph $4 \leq 2x - 6 < 12$. Write interval notation.



21. Solve $-5 < 3 - 2x \leq 11$.

22. Solve $x - 4 < -1$ or $2x + 3 \geq 13$.

23. Solve $3x + 1 \leq -8$ or $5x - 4 > 11$.

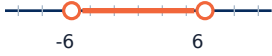
24. Explain why $x > 2$ and $x < -1$ has no solution, while $x > 2$ or $x < -1$ has two rays.

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E. Absolute-Value Inequalities

Objective: Interpret absolute value as distance and solve bounded or two-ray sets.

25. Solve and graph $|x| < 6$.



26. Solve $|x| \geq 4$.

27. Solve $|x-3| \leq 5$.

28. Solve $|2x+1| > 7$.

29. Solve $|3x-6| < 9$.

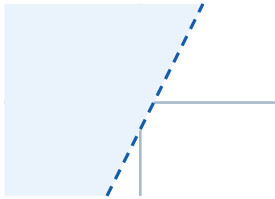
30. Solve $|x+4| = -2$ and explain the result.

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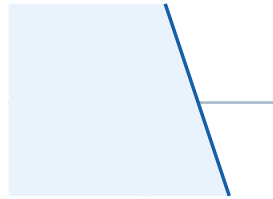
F. Inequalities in Two Variables

Objective: Graph boundary lines, choose line style, test points, and shade solution regions.

31. Graph $y > 2x - 1$ on the coordinate plane. State whether the boundary is solid or dashed and identify a test point.



32. Graph $3x + y \leq 6$. State the boundary equation, line style, and shaded side.



33. Determine whether (2,4) is a solution of $y \geq -x + 5$.

34. Determine whether (-1,3) satisfies $2x - 3y < 4$.

35. Write an inequality whose boundary is $y = -3x + 2$, dashed, with the origin in the solution region.

36. A graph uses a solid line $y = (1/2)x - 4$ and shades below it. Write the inequality.

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G. Modeling Constraints

Objective: Build and interpret inequalities involving capacity, averages, budgets, and dimensions.

37. A concert hall holds at most 850 people. If 638 seats are already reserved, write and solve an inequality for additional guests g .

38. A student needs an average of at least 80 on five tests. The first four scores are 74, 82, 79, and 88. Write and solve an inequality for the fifth score.

39. A delivery service charges \$9 plus \$2.50 per mile. A customer can spend no more than \$34. Write and solve an inequality for miles m .

40. A truck can carry at most 3,600 pounds. Crates weigh 145 pounds each and other cargo weighs 990 pounds. Find the greatest whole number of crates.

41. A phone battery has 92% charge and loses 7% per hour. For how many hours will it remain above 36%?

42. The perimeter of a rectangle is less than 70 cm. Its length is 4 cm more than twice width w . Find the possible positive widths.

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H. Advanced Inequality Analysis

Objective: Verify solutions and analyze identities, contradictions, and integer restrictions.

43. Solve $5-2(3x-4) \geq 4x+13$ and verify one solution and one nonsolution.

44. For what values of k does $kx+6 < kx+11$ hold for every real x ? Explain.

45. For what value of c does $3x+c \leq 3x-2$ have no solutions? Describe all such c .

46. Find all integers satisfying $|2x-1| \leq 7$ and list them.

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I. Reasoning, Proof, and Challenge

Objective: Correct reversal errors, prove the sign rule, determine parameters, and compare pricing plans.

47. Error analysis: A student changes $x < 4$ to $-x < -4$ after multiplying by -1 . Correct both the symbol and solution graph.

48. Prove that multiplying both sides of an inequality by a negative number reverses its direction.

49. Find k so that the solution of $2(kx-3) > 8x+4$ is $x < -5$. Verify your parameter.

50. A company offers Plan A: \$25 plus \$8 per class, and Plan B: \$61 plus \$5 per class. Determine when A costs less than B, when the costs are equal, and which plan is cheaper for 20 classes.