Name: ______________________

Date: ____________ Class: Algebra 1, per: ________

Answer Key: Chapter 6 - Balancing Inequalities

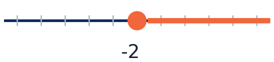
A. Inequality Language and Graphs

Objective: Translate inequalities and represent solution sets on number lines and with interval notation.

Directions: Show substitutions and simplification steps. Follow the order of operations, preserve units in applications, and explain all error-analysis, proof, and challenge responses.

Sign rule	**Compound forms**
Add/subtract: keep the symbol. Multiply/divide by a negative: reverse it.	$\lvert x\rvert<a \rightarrow -a<x<a$ $\lvert x\rvert>a \rightarrow x<-a$ or $x>a$

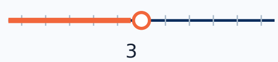
1. The graph has a closed point at -2 and shades right, so **$x\geq -2$**; interval **[-2,infinity)**.



2. 'At least' includes equality: **$x\geq -4$**; interval **[-4,infinity)**.

3. 'Below' means strictly less: **$T<18$**.

4. Use an open point at 3 and shade left. Interval: **(-infinity,3)**.



5. Integers include -2,-1,0,1,2,3. Any three, for example **-2,0,3**.

6. $x>5$ excludes 5 and uses an open point; $x\geq 5$ includes 5 and uses a closed point. Both shade right.

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B. One-Step Inequalities

Objective: Use inverse operations and reverse the symbol correctly for negative factors.

7. $x>7$. Open point at 7, shade right; **$(7,\text{infinity})$**. 	**8.** $y\le 7$; **$(-\text{infinity},7]$**.
9. $m<-7$; **$(-\text{infinity},-7)$**.	**10.** Divide by -4 and reverse: **$p\le -5$**. Reversal preserves the true order after multiplication/division by a negative.
11. Multiply by -3 and reverse: **$n>-21$**.	**12.** Dividing by -2 reverses > to <: **$x<-5$**. The graph has an open point at -5 and shades left.

Answer Key: Chapter 6 - Balancing Inequalities

C. Multi-Step Inequalities

Objective: Distribute, combine terms, clear denominators, and solve accurately.

13. $3x \le 21 \rightarrow$ **$x \le 7$**.	**14.** $-2y > 12 \rightarrow$ divide by -2 and reverse: **$y < -6$**.
15. $10m-5 \ge 3m+16 \rightarrow 7m \ge 21 \rightarrow$ **$m \ge 3$**.	**16.** $4-6x-15 < 10 \rightarrow -6x-11 < 10 \rightarrow -6x < 21 \rightarrow$ **$x > -7/2$**.
17. Multiply by 4: $x-2+2x+2 \ge 20 \rightarrow 3x \ge 20 \rightarrow$ **$x \ge 20/3$**.	**18.** $-1.8 < 1.8x+5.4 \rightarrow -7.2 < 1.8x \rightarrow$ **$x > -4$**.

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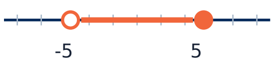
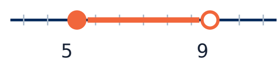
D. Compound Inequalities

Objective: Solve and graph intersections (and) and unions (or).

19. Subtract 2 throughout: $\mathbf{-5<x\le5}$; interval **(-5,5]**.



20. Add 6: $10\le2x<18$. Divide by 2: $\mathbf{5\le x<9}$; interval **[5,9)**.



21. Subtract 3: $-8<-2x\le8$. Divide by -2, reverse and reorder: $\mathbf{-4\le x<4}$.

22. First: $x<3$. Second: $2x\ge10 \rightarrow x\ge5$. Union: **(-infinity,3) union [5,infinity)**.

23. First: $3x\le-9 \rightarrow x\le-3$. Second: $5x>15 \rightarrow x>3$. Solution: **(-infinity,-3] union (3,infinity)**.

24. 'And' requires both conditions simultaneously, impossible here. 'Or' accepts either condition, giving **(-infinity,-1) union (2,infinity)**.

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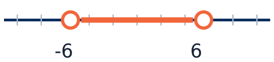
E. Absolute-Value Inequalities

Objective: Interpret absolute value as distance and solve bounded or two-ray sets.

25. Distance from 0 less than 6: **$-6<x<6$**; interval $(-6,6)$.



26. Distance at least 4: **$x\leq -4$ or $x\geq 4$**.

27. $-5\leq x-3\leq 5 \rightarrow$ **$-2\leq x\leq 8$**.

28. $2x+1>7$ or $2x+1<-7 \rightarrow x>3$ or $x<-4$. **(-infinity,-4) union (3,infinity)**.

29. $-9<3x-6<9 \rightarrow -3<3x<15 \rightarrow$ **$-1<x<5$**.

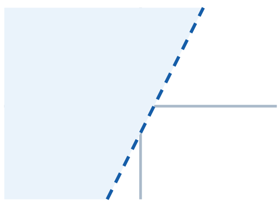
30. Absolute value is never negative, so **no solution**.

Answer Key: Chapter 6 - Balancing Inequalities

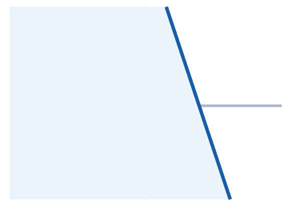
F. Inequalities in Two Variables

Objective: Graph boundary lines, choose line style, test points, and shade solution regions.

31. Boundary $y=2x-1$ is **dashed** because equality is excluded. Test (0,0): $0>-1$ is true, so shade the side containing the origin (above the line).



32. Boundary $\mathbf{y=-3x+6}$ is solid. Test (0,0): $0\le 6$ true, so shade the side containing the origin (below the line).



33. At $x=2$, right side$=-2+5=3$. Since $4\ge 3$, **(2,4) is a solution**.

34. $2(-1)-3(3)=-2-9=-11<4$, so **yes**.

35. At origin, 0 compared with 2: $0<2$. Dashed boundary means strict inequality, so one valid answer is $\mathbf{y<-3x+2}$.

36. Solid means equality included; below means y is less: $\mathbf{y\le(1/2)x-4}$.

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G. Modeling Constraints

Objective: Build and interpret inequalities involving capacity, averages, budgets, and dimensions.

37. $638+g\le 850 \rightarrow$ **$g\le 212$**, with $g\ge 0$ and whole. At most 212 additional guests.	**38.** $(74+82+79+88+x)/5\ge 80$. Sum first four=323, so $323+x\ge 400 \rightarrow$ **$x\ge 77$**.
39. $9+2.5m\le 34 \rightarrow 2.5m\le 25 \rightarrow$ **$m\le 10$ miles**, $m\ge 0$.	**40.** $145c+990\le 3600 \rightarrow 145c\le 2610 \rightarrow c\le 18$. Greatest whole number: **18 crates**.
41. $92-7t>36 \rightarrow -7t>-56 \rightarrow$ **$t<8$ hours**. It is above 36% before 8 hours; at 8 it equals 36%.	**42.** Length=$2w+4$. $P=2w+2(2w+4)<70 \rightarrow 6w+8<70 \rightarrow w<31/3$. With positive width: **$0<w<31/3$ cm**.

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H. Advanced Inequality Analysis

Objective: Verify solutions and analyze identities, contradictions, and integer restrictions.

43. $5-6x+8 \geq 4x+13 \rightarrow 13-6x \geq 4x+13 \rightarrow -10x \geq 0 \rightarrow$ **$x \leq 0$**. Check $x=-1$: $19 \geq 9$ true. Check $x=1$: $7 \geq 17$ false.	**44.** Subtract kx: $6<11$, always true. Thus **every real k** works.
45. Subtract $3x$: $c \leq -2$. This is true for all x when $c \leq -2$; it is false for all x (no solutions) when **$c>-2$**.	**46.** $-7 \leq 2x-1 \leq 7 \rightarrow -6 \leq 2x \leq 8 \rightarrow -3 \leq x \leq 4$. Integers: **-3,-2,-1,0,1,2,3,4**.

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I. Reasoning, Proof, and Challenge

Objective: Correct reversal errors, prove the sign rule, determine parameters, and compare pricing plans.

47. Multiplying $x<4$ by -1 gives $\mathbf{-x>-4}$. Equivalently $x<4$: open point at 4, shade left.	**48.** If $a<b$, then $b-a>0$. Multiply by $c<0$: $c(b-a)<0$, so $cb-ca<0$ and $cb<ca$; therefore $\mathbf{ca>cb}$. The order reverses.
49. $2kx-6>8x+4 \rightarrow (2k-8)x>10$. To obtain $x<-5$, the coefficient must be negative and $10/(2k-8)=-5$. Thus $2k-8=-2 \rightarrow \mathbf{k=3}$. Then $-2x>10 \rightarrow x<-5$.	**50.** A<B: $25+8c<61+5c \rightarrow 3c<36 \rightarrow \mathbf{c<12}$. Equal at $\mathbf{c=12}$. For $c>12$, B is cheaper. At 20 classes, A=\$185 and B=\$161, so **Plan B**.