Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can verify, represent, classify, and solve systems using precise algebraic reasoning.

For Questions 1-8, show the algebra or justification requested.

1. Verify whether $(4, 1)$ solves $\begin{cases} 2x + y = 9 \\ x - 3y = 1 \end{cases}$. Show both substitutions.	**2.** Find k so that $(-2, 5)$ satisfies $kx + 3y = 11$.
3. Two numbers have sum 26 and difference 8. Define variables and write a system.	**4.** Rewrite $3x - 2y = 12$ in slope-intercept form using exact notation.
5. Without finding an ordered pair, classify $\begin{cases} y = 2x - 5 \\ 4x - 2y = 10 \end{cases}$ as one, none, or infinitely many solutions. Explain.	**6.** Without solving for x and y, classify $\begin{cases} x + 2y = 7 \\ 2x + 4y = 19 \end{cases}$. Explain algebraically.
7. Choose the more efficient method for $\begin{cases} 5x + y = 8 \\ 3x - y = 4 \end{cases}$ and justify your choice. Then solve.	**8.** The equations $x + y = 9$ and $x - y = 1$ share the solution $(5, 4)$. Write a third, non-equivalent linear equation that also contains $(5, 4)$.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can solve systems by substitution and express fractional answers exactly.

For Questions 9-16, solve algebraically and check each ordered pair in one original equation.

9. Solve $\begin{cases} y = 3x - 7 \\ x + y = 9 \end{cases}$.	**10.** Solve $\begin{cases} x = 2y + 1 \\ 3x - y = 17 \end{cases}$.
11. Solve $\begin{cases} 2x + y = 11 \\ y = x - 1 \end{cases}$.	**12.** Solve $\begin{cases} y = -2x + 6 \\ 5x + 2y = 9 \end{cases}$.
13. Solve $\begin{cases} x + 3y = -5 \\ x = 4 - y \end{cases}$.	**14.** Solve $\begin{cases} 0.4x + 0.2y = 2.6 \\ y = x - 1 \end{cases}$. Give exact fractional answers.
15. Solve $\begin{cases} \dfrac{x-2}{3} + y = 4 \\ y = x - 4 \end{cases}$.	**16.** Solve $\begin{cases} y = \dfrac{1}{2}x + 4 \\ 3x - 2y = 8 \end{cases}$.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can solve systems by elimination, including equations with fractions and decimals.

For Questions 17-24, show the multiplier used whenever coefficients do not cancel immediately.

17. Solve $\begin{cases} 2x + 3y = 13 \\ 4x - 3y = 5 \end{cases}$.	**18.** Solve $\begin{cases} 5x - 2y = 4 \\ 3x + 2y = 12 \end{cases}$.
19. Solve $\begin{cases} 3x + 4y = -5 \\ 5x - 4y = 21 \end{cases}$.	**20.** Solve $\begin{cases} 0.2x + 0.5y = 2.9 \\ 0.3x - 0.2y = 0.4 \end{cases}$. Give exact answers.
21. Clear denominators, then solve $\begin{cases} \frac{x}{2} + \frac{y}{3} = 4 \\ \frac{x}{4} - \frac{y}{6} = 1 \end{cases}$.	**22.** Solve $\begin{cases} -4x + 3y = 18 \\ 6x - 5y = -28 \end{cases}$.
23. Classify $\begin{cases} 7x + 2y = 3 \\ 14x + 4y = 6 \end{cases}$ and justify.	**24.** Classify $\begin{cases} 3x - 6y = 12 \\ x - 2y = 5 \end{cases}$ and justify.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can analyze parameters, special systems, and algebraic structure.

For Questions 25-30, use proportionality, elimination, or substitution as appropriate.

25. For $\begin{cases} kx + 2y = 6 \\ 3x + 6y = 18 \end{cases}$, determine all k giving one solution, no solution, or infinitely many solutions.	**26.** Find a so that $\begin{cases} ax + 4y = 10 \\ 3x + 6y = 12 \end{cases}$ has no solution. Explain why.
27. Find m so that $(2, -1)$ is a solution of $mx - 3y = 11$.	**28.** Solve $\begin{cases} x + y = p \\ x - y = q \end{cases}$ for x and y in terms of p and q.
29. Two consecutive positive integers have sum 71. Write and solve a system to find them.	**30.** A two-digit number has digit sum 11 and is 27 greater than the number formed by reversing its digits. Write and solve a system.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can solve and interpret systems of linear inequalities using substitution and logical comparison.

Show each substitution and combine the resulting inequalities carefully.

31. Determine whether $(3, 2)$ satisfies all constraints: $2x + y \leq 9$, $x - y \geq 1$, $x \geq 0$, $y \geq 0$.

32. Determine whether $(-1, 4)$ satisfies $x + 2y \leq 7$, $x \geq 0$, and $y \geq 0$. Identify every failed constraint.

33. For $x = 4$, determine every value of y satisfying $y \leq 2x+1$ and $y \geq x - 2$. Express the result as a compound inequality.

34. For $y = 3$, determine every value of x satisfying $y \geq 2x-5$ and $y < -x + 7$.

35. Determine whether $x + y < 5$ and $x + y \geq 5$ can hold simultaneously. Justify your conclusion symbolically.

36. For $y = 2$, solve the system $3x + y \leq 14$ and $x - y > 1$ for x.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can create and use systems of constraints in realistic situations.

Define variables, write every required constraint, and retain integer restrictions when quantities are discrete.

37. Notebooks cost $\$3$ and binders cost $\$5$. A tutor has at most $\$60$ and needs at least 14 items. Write a complete system of constraints.	**38.** Using the constraints from Question 37, determine whether buying 8 notebooks and 6 binders is feasible.
39. Find all nonnegative integer pairs satisfying $x + y = 18$ and $x \geq 2y$. Show how you know the list is complete.	**40.** Desks seat 2 students and tables seat 4. At least 36 seats are needed, using at most 12 pieces of furniture. Write the constraints and test $(6, 6)$.
41. A school sold 180 tickets. Adult tickets cost $\$12$, student tickets cost $\$8$, and revenue was $\$1840$. Write and solve a system.	**42.** A jar contains 46 quarters and dimes worth $\$7.90$. Write and solve a system to find each number of coins.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can model mixtures, geometry, pricing, and sales with systems.

For Questions 43-48, define variables and interpret each result in context.

43. How many liters of 30% solution and 70% solution make 20 liters of a 46% solution? Write and solve a system.	**44.** A rectangle has perimeter 82 m. Its length is 5 m more than twice its width. Write and solve a system.
45. A school sold 420 event tickets. Adult tickets cost $\$10$, child tickets $\$6$, and revenue was $\$3320$. Find each ticket count.	**46.** Plan A costs $25 + 0.08m$ dollars and Plan B costs $13 + 0.12m$ dollars. Find the break-even usage m and common cost.
47. A student solves $\begin{cases} y = 2x + 1 \\ 3x - y = 7 \end{cases}$ by writing $3x - 2x + 1 = 7$. Identify the error and find the correct solution.	**48.** Prove that $\begin{cases} ax + by = c \\ ax + by = d \end{cases}$ has no solution whenever $c \neq d$.

Chapter 7 - Systems of Linear Equations and Inequalities

Objective: I can justify general claims and solve advanced elimination and parameter challenges.

Write a complete argument. Numerical answers without justification are incomplete.

49. Classify $\begin{cases} kx + 2y = 4 \\ 3x + (k+1)y = 7 \end{cases}$ for all real k: identify values giving one, none, or infinitely many solutions.	**50.** Solve by elimination: $\begin{cases} x + y + z = 12 \\ 2x - y + z = 7 \\ x + 2y - z = 6 \end{cases}$ Show two independent elimination steps.

Other topics for review:

- A solution satisfies every equation or inequality in the system.
- Substitution is efficient when one variable is isolated; elimination is efficient when coefficients cancel.
- Proportional coefficient rows distinguish dependent and inconsistent systems.
- Clear fractions and decimals before elimination, but report exact final values.
- Contextual systems may require nonnegative and integer restrictions.