Answer Key: Chapter 8 - The World of Polynomials

Objective: I can classify polynomials and identify their structural features using standard form.

Terms are assumed to have real coefficients. Equivalent terminology is acceptable.

1. For $7x^4 - 3x^2 + 5$: degree 4; leading term $7x^4$; leading coefficient 7; constant term 5; three nonzero terms, so it is a quartic trinomial.

2. For $-2x^3 + x - 9$: degree 3; leading term $-2x^3$; leading coefficient -2; constant term -9; three nonzero terms, so it is a cubic trinomial.

3. Arrange powers in descending order:

$$6 - 4x^5 + x^2 = \boxed{-4x^5 + x^2 + 6}.$$

The degree is 5, the leading coefficient is -4, and there are three nonzero terms.

4. A polynomial may contain only nonnegative integer exponents of its variables. Thus

$$\sqrt{x} = x^{1/2} \quad \text{and} \quad \frac{3}{x} = 3x^{-1}$$

are not polynomials. Only $\boxed{2x^3 - x + 1}$ is a polynomial.

5. Combine like terms:

$$4x^3 - 5x^3 = -x^3, \qquad 7 + 9 = 16.$$

Therefore the standard form is $\boxed{-x^3 + 3x^2 - 2x + 16}$.

6. The total degrees of the terms are $3 + 2 = 5$, $2 + 4 = 6$, and 0. The greatest is 6, so the polynomial has $\boxed{\text{degree } 6}$.

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Objective: I can evaluate polynomials and combine them by addition and subtraction.

7. For $p(t) = -\frac{1}{2}t^6 + 4t^3 - 9$: leading term $-\frac{1}{2}t^6$; leading coefficient $-\frac{1}{2}$; degree 6; constant term -9.

8. Many answers are possible. One example satisfying all conditions is

$$\boxed{-3x^4 + 2x^2 + 8}.$$

It has degree 4, leading coefficient -3, constant term 8, and exactly three nonzero terms.

9. Substitute $x = -2$:

$$P(-2) = 2(-2)^3 - 5(-2) + 1 = -16 + 10 + 1 = \boxed{-5}.$$

10. Substitute $a = -3$:

$$Q(-3) = -(-3)^4 + 3(-3)^2 - 7 = -81 + 27 - 7 = \boxed{-61}.$$

11. Combine corresponding terms:

$$(3x^2 - 2x + 5) + (-x^2 + 4x - 7) = \boxed{2x^2 + 2x - 2}.$$

12. Distribute the subtraction sign to every term:

$$(3x^2 - 2x + 5) - (-x^2 + 4x - 7) = 3x^2 - 2x + 5 + x^2 - 4x + 7 = \boxed{4x^2 - 6x + 12}.$$

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Objective: I can preserve signs and combine like terms accurately in polynomial operations.

13. Add like terms:

$$(5y^3 - 2y + 9) + (-3y^3 + 4y^2 + y - 6) = \boxed{2y^3 + 4y^2 - y + 3}.$$

14. Subtract term by term:

$$(8m^4 - m^2 + 3) - (2m^4 + 5m^2 - 7) = \boxed{6m^4 - 6m^2 + 10}.$$

15. Subtract the known addend from the sum:

$$R(x) = (5x^2 + x - 8) - (2x^2 - 3x + 4) = \boxed{3x^2 + 4x - 12}.$$

16. First,

$$P(-1) = (-1)^3 - 2(-1)^2 + 4 = 1,$$

and

$$Q(2) = 3(2)^2 - 2 - 5 = 5.$$

Thus $P(-1) + Q(2) = \boxed{6}$.

17. Distribute $4x^3$ and add exponents when multiplying like bases:

$$4x^3(-2x^4 + 5x - 7) = \boxed{-8x^7 + 20x^4 - 28x^3}.$$

18. Distribute $-3a^2b$:

$$-3a^2b(2ab^3 - 4a^2b + 5) = \boxed{-6a^3b^4 + 12a^4b^2 - 15a^2b}.$$

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Objective: I can multiply binomials and polynomials and report products in standard form.

19. Use distribution:

$$(x+5)(x-3) = x^2 - 3x + 5x - 15 = \boxed{x^2 + 2x - 15}.$$

20. Expand and combine:

$$(2x-1)(x+6) = 2x^2 + 12x - x - 6 = \boxed{2x^2 + 11x - 6}.$$

21. Expand:

$$(3y+4)(2y-5) = 6y^2 - 15y + 8y - 20 = \boxed{6y^2 - 7y - 20}.$$

22. Distribute $x+3$:

$$\begin{aligned}(x^2+2x-1)(x+3) &= x^3 + 3x^2 + 2x^2 + 6x - x - 3 \\ &= \boxed{x^3 + 5x^2 + 5x - 3}.\end{aligned}$$

23. Expand:

$$\begin{aligned}(2m^2 - m + 3)(m-2) &= 2m^3 - 4m^2 - m^2 + 2m + 3m - 6 \\ &= \boxed{2m^3 - 5m^2 + 5m - 6}.\end{aligned}$$

24. Area equals length times width:

$$(3x+2)(2x-5) = 6x^2 - 15x + 4x - 10 = \boxed{6x^2 - 11x - 10}.$$

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Objective: I can apply special-product identities and verify polynomial identities by expansion.

25. Use $(a+b)^2 = a^2 + 2ab + b^2$:

$$(x+7)^2 = \boxed{x^2 + 14x + 49}.$$

26. Use $(a-b)^2 = a^2 - 2ab + b^2$:

$$(3x-2)^2 = \boxed{9x^2 - 12x + 4}.$$

27. Use $(a+b)(a-b) = a^2 - b^2$:

$$(5y+1)(5y-1) = \boxed{25y^2 - 1}.$$

28. Square each term and include the middle product:

$$(2a+3b)^2 = 4a^2 + 12ab + 9b^2.$$

29. Compare $(x+k)^2 = x^2 + 2kx + k^2$ with $x^2 + 12x + 36$. Since $2k = 12$, $\boxed{k = 6}$; indeed $k^2 = 36$.

30. Expand or use difference of squares:

$$(x+4)^2 - (x-4)^2 = (2 \cdot 4)(2x) = \boxed{16x}.$$

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Objective: I can use polynomial identities and divide polynomials by monomials or binomials.

31. Distribute:

$$\begin{aligned}(x+2)(x^2-2x+4) &= x^3-2x^2+4x+2x^2-4x+8\\ &= \boxed{x^3+8}.\end{aligned}$$

32. Distribute:

$$\begin{aligned}(2x-3)(4x^2+6x+9) &= 8x^3+12x^2+18x-12x^2-18x-27\\ &= \boxed{8x^3-27}.\end{aligned}$$

33. Divide every term by $6x^2$:

$$\frac{18x^7-12x^4+6x^2}{6x^2} = \boxed{3x^5-2x^2+1}.$$

34. Divide coefficients and subtract exponents:

$$\frac{-15a^6b^3+10a^4b^2-5a^2b}{5a^2b} = \boxed{-3a^4b^2+2a^2b-1}.$$

35. Synthetic division by $x+4$ uses -4:

$$\begin{array}{r|rrrr} -4 & 1 & 5 & 2 & -8 \\ & & -4 & -4 & 8 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

The quotient is $\boxed{x^2+x-2}$.

36. Synthetic division by $x-3$ uses 3:

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

The quotient is $\boxed{2x^2+3x-2}$.

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Objective: I can apply the remainder and factor theorems and build polynomial models.

37. By the Remainder Theorem, the remainder on division by $x-2$ is

$$P(2) = 2(2)^3 - (2)^2 + 4(2) - 5 = 16 - 4 + 8 - 5 = \boxed{15}.$$

38. For $x+1$ to be a factor, $P(-1) = 0$:

$$(-1)^3 + k(-1)^2 - 4(-1) - 4 = 0 \Longrightarrow -1 + k + 4 - 4 = 0.$$

Thus $\boxed{k = 1}$.

39. Perimeter is twice the sum of length and width:

$$\begin{aligned} P &= 2(4x^2 + x - 3) + 2(x^2 - 2x + 5) \\ &= \boxed{10x^2 - 2x + 4}. \end{aligned}$$

40. Multiply the dimensions:

$$\begin{aligned} V &= x(x+2)(x+5) \\ &= x(x^2 + 7x + 10) \\ &= \boxed{x^3 + 7x^2 + 10x}. \end{aligned}$$

41. Profit is revenue minus cost:

$$\begin{aligned} P(x) &= 80x - (2x^2 + 35x + 120) \\ &= \boxed{-2x^2 + 45x - 120}. \end{aligned}$$

42. The border area equals outer area minus inner area:

$$(x+4)^2 - x^2 = x^2 + 8x + 16 - x^2 = \boxed{8x + 16}.$$

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Objective: I can evaluate polynomial models and analyze reasoning errors in polynomial operations.

43. Substitute $t = 2$:

$$h(2) = -16(2)^2 + 64(2) + 5 = -64 + 128 + 5 = \boxed{69 \text{ ft}}.$$

44. Area:

$$A(x) = (3x-2)^2 = \boxed{9x^2 - 12x + 4}.$$

At $x = 4$, the side is 10, so $A(4) = \boxed{100 \text{ square units}}$.

45. Expand the left side:

$$\begin{aligned}(x+1)(x^2-x+1) &= x^3 - x^2 + x + x^2 - x + 1\\ &= x^3 + 1.\end{aligned}$$

Therefore the identity is true for every real x.

46. The constant product must be $(-3)(4) = -12$, not $+12$:

$$(2x-3)(x+4) = 2x^2 + 8x - 3x - 12 = \boxed{2x^2 + 5x - 12}.$$

47. When adding like terms, coefficients are added but exponents do not change:

$$(3x^2 + 2x - 1) + (x^2 - 5x + 4) = \boxed{4x^2 - 3x + 3}.$$

The incorrect $4x^4$ came from adding exponents during addition.

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Objective: I can prove identities and determine unknown polynomial coefficients from conditions.

48. Expand:

$$(x^2+1)^2 = x^4 + 2x^2 + 1.$$

Thus $x^4 + 2x^2 + 1$ and $(x^2+1)^2$ are identical for all real x.

49. Since $P(x) = ax^2 + bx + c$ and $P(0) = 3$, $c = 3$. The other conditions give

$$a + b + 3 = 8 \implies a + b = 5,$$

$$a - b + 3 = 2 \implies a - b = -1.$$

Adding yields $2a = 4$, so $a = 2$ and $b = 3$. Therefore

$$\boxed{P(x) = 2x^2 + 3x + 3}.$$

50. Let $A = (x+1)^2$ and $B = (x-1)^2$. Then

$$(x+1)^4 - (x-1)^4 = A^2 - B^2 = (A-B)(A+B).$$

Now $A - B = 4x$ and $A + B = 2x^2 + 2$. Hence

$$(4x)(2x^2+2) = \boxed{8x^3 + 8x}.$$

Key review principles: write in descending powers; combine only like terms; distribute every sign; add exponents only when multiplying like bases; and use identities to shorten reliable algebra.