Answer Key: Chapter 9 - The Art of Factoring

Objective: I can factor out greatest common factors and factor polynomial expressions by grouping.

Each answer is factored completely over the integers unless stated otherwise.

1. The greatest common factor of $12x^3$ and $18x^2$ is $6x^2$:

$$12x^3 + 18x^2 = \boxed{6x^2(2x+3)}.$$

2. The numerical GCF is 10, and the shared variable factor is a^2b:

$$20a^4b^2 - 30a^3b^3 + 10a^2b = \boxed{10a^2b(2a^2b - 3ab^2 + 1)}.$$

3. Factor out a negative GCF so the leading term inside is positive:

$$-8m^5 + 12m^3 - 4m^2 = \boxed{-4m^2(2m^3 - 3m + 1)}.$$

4. The GCF is $7xy$:

$$14x^2y - 21xy^2 = \boxed{7xy(2x - 3y)}.$$

5. The common binomial is $x+5$:

$$x(x+5) + 3(x+5) = \boxed{(x+5)(x+3)}.$$

6. Group the first two and last two terms:

$$\begin{aligned} x^3 + 4x^2 + 3x + 12 &= x^2(x+4) + 3(x+4) \\ &= \boxed{(x+4)(x^2+3)}. \end{aligned}$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can use grouping and factor monic quadratic trinomials.

7. Group:

$$\begin{aligned} 2x^3 - 6x^2 + 5x - 15 &= 2x^2(x-3) + 5(x-3) \\ &= \boxed{(x-3)(2x^2+5)}. \end{aligned}$$

8. The GCF of all terms is $3pq$:

$$9p^2q - 15pq^2 + 6pq = \boxed{3pq(3p - 5q + 2)}.$$

9. Find two integers with product 12 and sum 7: 3 and 4.

$$x^2 + 7x + 12 = \boxed{(x+3)(x+4)}.$$

10. Find two integers with product -20 and sum -1: -5 and 4.

$$x^2 - x - 20 = \boxed{(x-5)(x+4)}.$$

11. The required integers are -3 and -8:

$$x^2 - 11x + 24 = \boxed{(x-3)(x-8)}.$$

12. The required integers are 7 and -5:

$$x^2 + 2x - 35 = \boxed{(x+7)(x-5)}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can factor monic and nonmonic quadratic trinomials accurately.

13. Two integers with product 20 and sum -9 are -4 and -5:

$$y^2 - 9y + 20 = \boxed{(y-4)(y-5)}.$$

14. Two integers with product 40 and sum 13 are 5 and 8:

$$a^2 + 13a + 40 = \boxed{(a+5)(a+8)}.$$

15. Two integers with product -24 and sum 5 are 8 and -3:

$$x^2 + 5x - 24 = \boxed{(x+8)(x-3)}.$$

16. This is a perfect-square trinomial:

$$x^2 - 12x + 36 = \boxed{(x-6)^2}.$$

17. Since $ac = 6$, use 6 and 1:

$$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = \boxed{(2x+1)(x+3)}.$$

18. Since $ac = -12$, use -12 and 1:

$$3x^2 - 11x - 4 = 3x^2 - 12x + x - 4 = \boxed{(3x+1)(x-4)}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can factor nonmonic trinomials by decomposition and grouping.

19. Since $ac = -12$, use 4 and -3:

$$6x^2 + x - 2 = 6x^2 + 4x - 3x - 2 = \boxed{(3x+2)(2x-1)}.$$

20. Since $ac = -60$, use 6 and -10:

$$4x^2 - 4x - 15 = 4x^2 + 6x - 10x - 15 = \boxed{(2x+3)(2x-5)}.$$

21. Since $ac = 30$, use 10 and 3:

$$5y^2 + 13y + 6 = 5y^2 + 10y + 3y + 6 = \boxed{(5y+3)(y+2)}.$$

22. Since $ac = 24$, use -12 and -2:

$$8a^2 - 14a + 3 = 8a^2 - 12a - 2a + 3 = \boxed{(4a-1)(2a-3)}.$$

23. Since $ac = -30$, use 6 and -5:

$$10m^2 + m - 3 = 10m^2 + 6m - 5m - 3 = \boxed{(5m+3)(2m-1)}.$$

24. Since $ac = -120$, use 8 and -15:

$$12x^2 - 7x - 10 = 12x^2 + 8x - 15x - 10 = \boxed{(3x+2)(4x-5)}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can recognize and apply special factoring identities.

25. Use $a^2 - b^2 = (a-b)(a+b)$:

$$x^2 - 81 = \boxed{(x-9)(x+9)}.$$

26. Both terms are perfect squares:

$$16x^2 - 25y^2 = \boxed{(4x-5y)(4x+5y)}.$$

27. The middle term is $2(3a)(5) = 30a$:

$$9a^2 + 30a + 25 = \boxed{(3a+5)^2}.$$

28. The middle term is $-2(2m)(5) = -20m$:

$$4m^2 - 20m + 25 = \boxed{(2m-5)^2}.$$

29. Use $a^3 - b^3 = (a-b)(a^2+ab+b^2)$:

$$x^3 - 27 = \boxed{(x-3)(x^2+3x+9)}.$$

30. Use $a^3 + b^3 = (a+b)(a^2-ab+b^2)$:

$$8y^3 + 125 = \boxed{(2y+5)(4y^2-10y+25)}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can factor sums or differences of cubes and factor mixed expressions completely.

31. Use a difference of cubes:

$$64a^3 - b^3 = \boxed{(4a - b)(16a^2 + 4ab + b^2)}.$$

32. Use a sum of cubes:

$$27x^3 + 8 = \boxed{(3x + 2)(9x^2 - 6x + 4)}.$$

33. First factor the GCF, then the difference of squares:

$$3x^3 - 27x = 3x(x^2 - 9) = \boxed{3x(x - 3)(x + 3)}.$$

34. Factor the GCF and recognize a perfect square:

$$2x^3 + 8x^2 + 8x = 2x(x^2 + 4x + 4) = \boxed{2x(x + 2)^2}.$$

35. Factor repeatedly:

$$\begin{aligned} 5x^4 - 80 &= 5(x^4 - 16) \\ &= 5(x^2 - 4)(x^2 + 4) \\ &= \boxed{5(x - 2)(x + 2)(x^2 + 4)}. \end{aligned}$$

36. Factor the GCF and then the trinomial:

$$6x^3 - 24x^2 + 24x = 6x(x^2 - 4x + 4) = \boxed{6x(x - 2)^2}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can factor higher-degree expressions completely and solve equations by the Zero Product Property.

37. Apply difference of squares twice:

$$a^4 - 16 = (a^2 - 4)(a^2 + 4) = \boxed{(a-2)(a+2)(a^2+4)}.$$

38. Treat x^2 as one variable:

$$\begin{aligned} x^4 - 5x^2 + 4 &= (x^2 - 1)(x^2 - 4) \\ &= \boxed{(x-1)(x+1)(x-2)(x+2)}. \end{aligned}$$

39. Factor and set each factor equal to zero:

$$x^2 + 7x + 12 = (x+3)(x+4) = 0.$$

Thus $\boxed{x = -3, -4}$.

40. Factor:

$$2x^2 - 5x - 3 = (2x+1)(x-3) = 0.$$

Therefore $\boxed{x = -\frac{1}{2},\ 3}$.

41. Move all terms to one side:

$$3x^2 - 27 = 3(x^2 - 9) = 3(x-3)(x+3) = 0.$$

Thus $\boxed{x = -3,\ 3}$.

42. Factor completely:

$$x^3 - 4x = x(x^2 - 4) = x(x-2)(x+2) = 0.$$

Therefore $\boxed{x = -2,\ 0,\ 2}$.

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can solve polynomial equations, interpret factors in context, and analyze common factoring errors.

43. Factor:

$$4x^2 - 12x + 9 = (2x-3)^2 = 0.$$

The repeated solution is $\boxed{x = \frac{3}{2}}$.

44. Group:

$$\begin{aligned} x^3 + 2x^2 - 9x - 18 &= x^2(x+2) - 9(x+2) \\ &= (x+2)(x^2-9) \\ &= (x+2)(x-3)(x+3). \end{aligned}$$

Thus $\boxed{x = -3, -2, 3}$.

45. Find two positive constants with product 24 and sum 11: 3 and 8. The dimensions may be

$$\boxed{x+3 \quad \text{and} \quad x+8}.$$

46. Factor the area:

$$6x^2 - 7x - 3 = (3x+1)(2x-3).$$

If one dimension is $3x+1$, the other is $\boxed{2x-3}$.

47. The proposed factors give

$$(x-1)(x-6) = x^2 - 7x + 6,$$

so the middle coefficient is wrong. Numbers with product 6 and sum -5 are -2 and -3:

$$\boxed{x^2 - 5x + 6 = (x-2)(x-3)}.$$

Answer Key: Chapter 9 - The Art of Factoring

Objective: I can justify irreducibility, determine parameters, and factor challenging identities.

48. The product $(x-5)(x+5)$ equals x^2-25, not x^2+25. Since $x^2+25>0$ for every real x, it has no real zeros and no real linear factors. It is $\boxed{\text{irreducible over the real numbers}}$.

49. If the positive constant factors are consecutive and have product 20, they must be 4 and 5:

$$(x+4)(x+5)=x^2+9x+20.$$

Therefore $\boxed{k=9}$.

50. Create a difference of squares:

$$\begin{aligned} x^4+4x^2+16 &= (x^2+4)^2-(2x)^2 \\ &= \boxed{(x^2+2x+4)(x^2-2x+4)}. \end{aligned}$$

Neither quadratic factors further over the integers.

Key review principles: remove the GCF first; verify factor pairs by product and sum; recognize special identities; continue until every factor is irreducible over the stated number system; and check by multiplication.