

Answer Key: Chapter 4 - Trigonometric Identities

Objective: Solutions are checked for mathematical accuracy. Algebraically equivalent steps are acceptable where expressions are defined.

1. Answer: $\tan x = \frac{\sin x}{\cos x}$

Work: This is the quotient identity for tangent.

2. Answer: $\cot x = \frac{\cos x}{\sin x}$

Work: This is the quotient identity for cotangent.

3. Answer: $\sec x = \frac{1}{\cos x}$

Work: This is the reciprocal identity for secant.

4. Answer: $\csc x = \frac{1}{\sin x}$

Work: This is the reciprocal identity for cosecant.

5. Answer: 1

Work: Fundamental Pythagorean identity, obtained from $x^2 + y^2 = r^2$ after division by r^2 .

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6. Answer: $\sec^2 x$

Work: Divide $\sin^2 x + \cos^2 x = 1$ by $\cos^2 x$.

7. Answer: $\csc^2 x$

Work: Divide $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$.

8. Answer: 1

Work: Rearrange $1 + \tan^2 x = \sec^2 x$.

9. Answer: 1

Work: Rearrange $1 + \cot^2 x = \csc^2 x$.

10. Answer: 1

Work: $\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 1$.

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11. Answer: 1

Work: $\frac{1}{\cos x} \cos x = 1.$

12. Answer: 1

Work: $\frac{1}{\sin x} \sin x = 1.$

13. Answer: $\sin x$

Work: $\frac{\sin/\cos}{1/\cos} = \sin x.$

14. Answer: $\cos x$

Work: $\frac{\cos/\sin}{1/\sin} = \cos x.$

15. Answer: $\sec x$

Work: $\frac{\sin}{\cos} \cdot \frac{1}{\sin} = \frac{1}{\cos} = \sec x.$

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16. Answer: $\csc x$

Work: $\frac{\cos}{\sin} \cdot \frac{1}{\cos} = \frac{1}{\sin} = \csc x.$

17. Answer: $\tan x$

Work: $\sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x}.$

18. Answer: $\cot x$

Work: $\cos x \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x}.$

19. Answer: 1

Work: Difference of squares gives $\sec^2 x - \tan^2 x = 1.$

20. Answer: 1

Work: Difference of squares gives $\csc^2 x - \cot^2 x = 1.$

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21. Answer: $\sin x$

Work: Replace $1 - \cos^2 x$ by $\sin^2 x$ and cancel one $\sin x$.

22. Answer: $\cos x$

Work: Replace $1 - \sin^2 x$ by $\cos^2 x$ and cancel.

23. Answer: $\tan x$

Work: Use $\sec^2 x - 1 = \tan^2 x$.

24. Answer: $\cot x$

Work: Use $\csc^2 x - 1 = \cot^2 x$.

25. Answer: $\sin x$

Work: $\sec x - \cos x = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$; divide by $\frac{\sin x}{\cos x}$.

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26. Answer: $\cos x$

Work: $\csc x - \sin x = \frac{\cos^2 x}{\sin x}$; divide by $\frac{\cos x}{\sin x}$.

27. Answer: $1 - \cos x$

Work: Factor $\sin^2 x = 1 - \cos^2 x = (1 - \cos x)(1 + \cos x)$.

28. Answer: $1 - \sin x$

Work: Factor $\cos^2 x = (1 - \sin x)(1 + \sin x)$.

29. Answer: $\frac{\sin x}{1 + \cos x}$

Work: Multiply the left side by $\frac{1 + \cos x}{1 + \cos x}$; numerator becomes $\sin^2 x$.

30. Answer: $\frac{\cos x}{1 + \sin x}$

Work: Multiply the left side by $\frac{1 + \sin x}{1 + \sin x}$; numerator becomes $\cos^2 x$.

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31. Answer: $\frac{1 + \sin x}{\cos x}$

Work: $\frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1 + \sin x}{\cos x}.$

32. Answer: $\frac{1 - \sin x}{\cos x}$

Work: Combine over the common denominator $\cos x$.

33. Answer: $\frac{1 + \cos x}{\sin x}$

Work: $\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1 + \cos x}{\sin x}.$

34. Answer: $\frac{1 - \cos x}{\sin x}$

Work: Combine over the common denominator $\sin x$.

35. Answer: $\sec x \csc x$

Work: $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}.$

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36. Answer: $\tan x + \cot x$

Work: Left side = $\frac{1}{\sin x \cos x}$; the right side combines to the same expression.

37. Answer: $\tan x + \cot x$

Work: Left side = $\frac{1}{\sin x \cos x}$; combine the right side over $\sin x \cos x$.

38. Answer: $\csc x + \cot x$

Work: Rationalize: $\frac{\sin x(1+\cos x)}{1-\cos^2 x} = \frac{1+\cos x}{\sin x}$.

39. Answer: $\sec x + \tan x$

Work: Rationalize to $\frac{1+\sin x}{\cos x}$.

40. Answer: $\csc x - \cot x$

Work: Rationalize to $\frac{1-\cos x}{\sin x}$.

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41. Answer: $\sec x - \tan x$

Work: Rationalize to $\frac{1-\sin x}{\cos x}$.

42. Answer: $2 \sin x \cos x$

Work: Use $\sin(x+x) = \sin x \cos x + \cos x \sin x$.

43. Answer: $\cos^2 x - \sin^2 x$

Work: Use $\cos(x+x) = \cos x \cos x - \sin x \sin x$.

44. Answer: $1 - 2 \sin^2 x$

Work: Start with $\cos^2 x - \sin^2 x$ and replace $\cos^2 x$ by $1 - \sin^2 x$.

45. Answer: $2 \cos^2 x - 1$

Work: Start with $\cos^2 x - \sin^2 x$ and replace $\sin^2 x$ by $1 - \cos^2 x$.

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46. Answer: $\cos x$

Work: Use $\sin(A - B) = \sin A \cos B - \cos A \sin B$ with $A = \frac{\pi}{2}$.

47. Answer: $\sin x$

Work: Use $\cos(A - B) = \cos A \cos B + \sin A \sin B$ with $A = \frac{\pi}{2}$.

48. Answer: $-\sin x$

Work: On the unit circle, reflection across the x -axis keeps cosine and reverses the sign of sine.

49. Answer: $\cos x$

Work: Unit-circle reflection across the x -axis leaves the x -coordinate unchanged.

50. Answer: $-\tan x$

Work: $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x}$.